OBSERVER BASED ROBUST ADAPTIVE TRACKING FOR UNCERTAIN ROBOT MANIPULATORS WITH EXTERNAL FORCE DISTURBANCE REJECTION

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SYSTEMS AND CONTROL ENGINEERING

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BY

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This thesis, written by **MOHAMMED ABDUL REHAN KHAN** under the direction of his thesis advisor and approved by his thesis committee, has been presented and accepted by the Dean of Graduate Studies, in partial fulfillment of the requirements for the degree of **MASTER OF SCIENCE IN SYSTEMS AND CONTROL ENGINEERING.**

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بسم الله الرحمن الرحيم

IN THE NAME OF ALLAH, THE MOST GRACIOUS, THE MOST MERCIFUL

Dedicated to

My Beloved Parents

ACKNOWLEDGMENTS

All praises and worship are for ALLAH SWT alone; we seek His help and ask for His forgiveness, we thank Him for providing us with knowledge and we always strive to achieve His reward. I feel privileged to glorify His name in the sincerest way through this small accomplishment. Peace and blessings be upon the greatest human being that ever walked on this earth, the last and the final Prophet, Muhammad (peace be upon him), upon his family, his companions, and all those who follow him until the day of judgement.

I would like to thank my parents, Mr. and Mrs. Abdul Rahman Khan, from the bottom of my heart for their immense patience and the motivation they had given me and also for teaching me the true meaning of hard work through their lives. My brother Mohammed Abdul Farhan Khan, my sisters, and my sweetheart Ms. Rida Khan, too share the same acknowledgements for their support. It would not be wrong to say that they have worked more than me, in making me what I am today. I ask Allah to reward all of them the best in this world and the hereafter, aameen.

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جزاكم الله خيرا

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LIST OF ABBREVIATIONS

AAR	Agile Anthropomorphic Robot.
AC	Adaptive Control
ACT	Adaptive Computed Torque.
ANL	Argonne National Laboratory.
ARM	Autonomous Robotic Manipulation.
BRA	British Robot Association.
CCS	Cooperative Control Scheme.
CHGO	Constant High Gain Observer.
DARPA	Defense Advanced Research Projects Agency.
DOF	Degree of Freedom.
HGO	High Gain Observer.
JIRA	Japan Industrial Robot Association.
LDO	Linear Disturbance Observer.
LS3	Legged Squad Support Systems.
MIMO	Multi-Input and Multi-Output.

MRAC	Model Reference Adaptive Controller.
PD	Proportional-Derivative.
PID	Proportional-Integral-Derivative.
RAC	Robust Adaptive Control.
RBD	Rigid Body Dynamics.
RIA	Robotics Institute of America
RLS	Recursive Least Square algorithm.
SMC	Sliding Mode Controller.
STR	Self-Tuning Regulator.
TVHGO	Time Varying High Gain Observer.
VSC	Variable Structure Control.
$q, \dot{q}, \ddot{q}$	Vector of joint angular position, angular velocity and angular acceleration.
$\hat{q}, \dot{\hat{q}}, \ddot{\hat{q}}$	Vector of estimated joint angular position, angular velocity and angular acceleration.
$q_d, \dot{q}_d, \ddot{q}_d$	Vector of desired joint angular position, angular velocity and angular acceleration.
$q_r, \dot{q}_r, \ddot{q}_r$	Vector of reference joint angular position, angular velocity and angular acceleration.

$e, \dot{e}, \ddot{e}$	Position error, velocity error and acceleration error.
M	Inertial matrix.
h	Coriolis, centripetal and gravitational forces matrix.
C	Coriolis and centripetal matrix.
J	Jacobian matrix.
$\mathrm{F}_{\mathrm{ext}}$	External force vector.
l	Length of the robot manipulator arm.
τ	Applied torque to be designed.
M_0	Nominal part of the inertial matrix.
M_Δ	Norm-bounded unmodeled part of inertial matrix.
h_0	Known vector of h matrix.
h_Δ	Vector of unstructured uncertainties.
S, S_r	Metric functions.
K	Positive definite matrix.
x	State vector of the state space control.
y	Output vector of the state space control.

$\hat{x}$	Estimate state vector of the state space control.
A	System matrix of the state space control.
B	Input matrix of the state space control.
C	Output matrix of the state space control.
L	Observer gain matrix.
V	Lyapunov functions.
$K_{d,i}, K_{p,i}$	Positive definite gain matrices.
K_{cp}, K_{cv}, K_{ca}	Positive semi-definite diagonal matrices.
$\forall$	For all.
$\in$	Belongs to.
$\Rightarrow$	Implies.
$\Leftrightarrow$	Equivalent to.
inf	Infimum.
min	Minimum.
$\mathcal{R}^n$	Space of real n-dimensional vectors.
$\mathcal{R}^{n\times n}$	Space of n by n matrices.

$\lvert\cdot\rvert$	Absolute value of a scalar.
$\lVert\cdot\rVert$	Euclidean norm of a vector.
Σ	A dynamic system.
Δ	Model uncertainty.
Λ	A positive definite diagonal matrix.
$\alpha_1, \alpha_2, \alpha_3$	Unknown constants.
Θ	Dimensionally compatible matrix.
η	Unknown parameter.
$\gamma_\alpha, \gamma_\eta$	Adaptation gains.
Γ	Adaptation matrix.
ϕ	Vector of uncertain parameters.
λ	Lagrange multiplier vector.
ψ	Unknown nonlinear dynamics.
ς	Stability factor.

ABSTRACT

Full Name : [Mohammed Abdul Rehan Khan]

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Robotics research in today’s era is mainly focused on multi-robot manipulator systems. Since a single robot manipulator is no longer the best solution for several applications such as robotic exploration in hazardous environments, automated production plants, space exploration, deep water energy exploration, operational or recovery missions in hostile environments, autonomous robot for operations in remote locations and agricultural purposes, trajectory planning of the robotic manipulator is significant in these applications. The performance of the manipulator in this endeavor is influenced by parametric, non-parametric uncertainties, unmodeled dynamics and external disturbances. These influences, in addition to operational errors, will also result in the reduced life span of the manipulator.

Several possible control strategies have been proposed in the literature. In this thesis, initially we propose an observer based robust adaptive control strategy with both time invariant and time-varying observer gains and then implement it to multiple robot manipulators. The objective is to minimize the undesirable disturbances and make the system follow a chosen reference model or trajectory. Lyapunov method is used to derive the tuning parameters and to ensure the stability of the proposed scheme. Firstly, simulation results carried on two degree of freedom robot manipulator shows that the

proposed robust adaptive control scheme achieves boundedness for all the closed-loop signals and ensures convergence of the tracking error, then this control law is applied to three identical two degree of freedom robot manipulator concluding that the proposed robust adaptive synchronization control scheme achieves boundedness for all the closed-loop signals and ensures convergence of both the tracking and synchronization errors.

ملخص الرسالة

الاسم الكامل: محمد عبدالريحان خان

عنوان الرسالة: التتبع المتكيف المُحكم لأذرع إنسان آلي غير محدد القيم بواسطة نبذ اضطرابات القوة الخارجية القائم على مراقب

التخصص: هندسة النظم والتحكم

تاريخ الدرجة العلمية: إبريل 2015

البحث فى مجال الآليات فى العصر الحالي يركز بشكل أساسي علي أنظمة الأذرع متعددة الآليات. حيث أن الذراع الآلي الأحادي لم يعد الحل الأمثل للعديد من التطبيقات مثل الألي الأستكشافي فى المنطاق الخطرة وخطوط الأنتاج المؤتمتة ومجال أستكشاف الفضاء وأستكشاف الطاقة فى المياة العميقة ومهمات التشغيل والأصلاح فى البيئات المعادية الخطرة وأيضا الآلي ذاتي التحكم لتنفيذ العمليات فى المناطق البعيدة وأخيراً فى الأغراض الزراعية. تخطيط مسار الذراع الآلي فى هذة التطبيقات يعد أمراً هاماً. يتأثر أداء الذراع فى تلك الاغراض بنسب الشك البارامترية والابارامترية والديناميكيات الغير منمذجة و أيضاً الأضطرابات الخارجية. هذة التأثيرات بالأضافة الى اخطاء التشغيل ستؤدي أيضاً الى تقليل العمر الأفتراضي للذراع.

العديد من أستراتيجيات التحكم الممكنة تم طرحها فى أستبيان الأبحاث السابقة. فى هذة الرسالة نطرح اولاً أستراتيجية التحكم المتكيف المحُكم القائم على مراقب مع كلاً من معاملات تضخيم المراقب المعتمدة والغير معتمدة على الزمن ثم تنفيذها على أذرع آلية متعددة. الغرض الأساسى هو تقليل الأضطرابات الغير مرغوب فيها وتمكين النظام من تعقب مسار أو نموذج ديناميكي مرجعي مُحدد ومُختار. تم أستخدام طريقة ليابونوف فى أستنتاج معاملات الموالفة وأيضاً ضمان استقرار المخطط المقترح. بداية أظهرت نتائج المحاكاة التي نُفذت على ذراع آلي ثنائي درجة الحرية ان مخطط المتحكم المتكيف المُحكم استطاع تقييد أشارات النظام المغلق وضمن القضاء على نسبة الخطأ فى نظام التتبع لتؤول الى الصفر ومن ثم تطبيق أشارة التحكم هذة على ثلاثة أذرع آلية متطابقة ذات درجة حرية ثنائية يفيد بأن

مخطط التحكم التزامني المتكيف المُحكم يستطيع تحقيق تلك القيود لجميع أشارات الأنظمة المغلقة ويضمن تتبع المرجع المحدد بالأضافة الى القضاء على نسبة الخطأ المتزامنة.

ماجستير في العلوم الهندسية

جامعة الملك فهد للبترول والمعادن

الظهران, المملكة العربية السعودية

2015

CHAPTER 1

INTRODUCTION

1.1 Review

The history of the human fascination with physically constructed life forms, which includes robots and automatic machines, is long. A historic overview of the fascination with regard to the robot evolution and its effects upon human life was presented by Mowforth [1]. He also discussed the growing expectations from the use of robots in contemporary industry. The industrial robot also called as robot manipulator was pioneered by George Duvall and Joe Englberger, who brought the first unimate to market in 1957 [2].

Robotic manipulators are used to perform a variety of different tasks. In factories they perform mechanical tasks repeatedly with accuracy, even in a hazardous and unhealthy environment caused for example, by dust, fumes, heat, radiation or noise. Because robot manipulators can perform with speed and consistency, they are considered to be good replacement for human beings. However, most industrial robots [3], found in the factory operate with simple independent-joint controllers, which makes difficult for robots to carry out more complex operations. They have yet to take full advantage of the recent advances in control theory [4] and computer technology. Because the simple controllers for present-day industrial robots do not compensate for the complicated effects of inertial, centrifugal, Coriolis, gravity, and friction forces when the robots are in motion they are mostly used in rather simple, repetitive tasks which tend not to require high precision. When the robot is required, for example, to follow a desired trajectory at high speed with small tracking errors, a simple controller fails to give satisfactory performance. There has been considerable interest in developing sophisticated control algorithms for robotic manipulators. Unfortunately, these algorithms are rarely found to be successfully used outside of research laboratories today.

To define a robotic manipulator a number of questions concerning the functional concepts, should be answered: What is, and what is not, a robot? How is a robot constructed? How does it operate? Despite this, only a few manufacturers and associations would agree on one single definition [5]. Since there is no standard definition, it would be helpful to consider some of the attempts in this regard:

1. The *British Robot Association* (BRA) emphasizes the four degrees of freedom as one of the qualifications defining a robot as: “A reprogrammable device with a minimum of four degrees of freedom designed to both manipulate and transport parts, tools or specialized manufacturing implements through variable programmed motions for the performance of the specific manufacturing task”.

2. The *Robotics Institute of America* (RIA) defines the robot as: “A reprogrammable multi-functional manipulator designed to move material, parts. Tools or specialized devices through variable programmed motions for the performance of a variety of tasks”. The RIA emphasizes the programmable facilities, and its definition is widely accepted for an industrial robot.

3. The *Japan Industrial Robot Association* (JIRA) and the Japanese Industrial Standards Committee define the robots at various levels as: Manipulator: “A machine which has functions similar to those of the human upper limbs, and moves the objects spatially, from one location to the other”. Playback robot: “A manipulator which is able to perform an operation by reading off the memorized information for an operating sequence. Including positions and the like, which it learned by being taken manually through the routine beforehand and base higher levels definitions upon the first one”.

Generally, a robotic manipulator is thought of as a programmable machine constructed by a chain of interconnected links by means of rotary or sliding joints as seen in figure 1. Where each joint can be actuated independently by its own actuator to allow the end effector to follow a defined trajectory in order to perform a defined task [6]. To attain the desired features a manipulator should be equipped with good sensors, a good control system with adequate computing power and light weight links. This is correct when developing new manipulators [7]. However, there is still the need to improve the qualities of the existing ones. This can be done through implementing more sophisticated control algorithms or adjusting the existing ones.

Whether developing new control algorithm for the manipulators or improving existing ones, a good model of the system is required [8]. An accurate mathematical model would be implemented in new control schemes such as computed torque control and model based control [9]. A fully inclusive dynamic model could be complex and computationally expensive if implemented in real time applications. Generating such models for robotic manipulators is difficult and error prone, despite the existence of adequate formulations such as the Newton-Euler and Lagrange formulations. Computer automatic model generation using multibody dynamic systems modelling packages is obviously desirable [10]. These do not support all the modelling activities required for robotic manipulators and may need further development. A good model is also required for computer simulation to predict the behavior of a particular manipulator under particular conditions of actuation. The simulation exercise aids the analysis of the manipulator design and performance evaluation, as well as the evaluation of controller design [11].

Although the dynamic model is critical for the above activities several simplifying assumptions and approximations are considered during its development [12]. Therefore, the model does not have to be inclusive of all characteristics to be a valid description of the manipulator dynamics, but it is valid if it is eventually judged fit for the purpose for which it was intended [13]. The end-effector positioning through off-line programming relies fundamentally on the manipulator internal functional relationship between the end-effector and the base. This relationship, the kinematic model, is unique for each manipulator and should be established accurately after manufacturing, as discussed by Roth et al [14]. Manipulators operating underwater are not different in kinematics from those operating in normal conditions, however their dynamics are severely affected by the hydrodynamic effects. A better understanding of their dynamics is required since there is an increased need for their use in underwater activities related to sea bed exploration, rescue and similar activities [15]. In spite the fact that generic models of underwater manipulators have been proposed, no study of the particular hydrodynamic effects on a specific manipulator model has been reported. This issue can be addressed as a disturbance added to the dynamic model of the system, where hydrodynamic effects are explicitly calculated [16], [17].

Although Adaptive Control (AC) presented in the literature, which can deal with structured uncertainty (a correct structural model with uncertain parameter values) as well as other unwanted disturbances, a finite time is required for achieving convergence of the tracking error to zero. This means that the performance at the beginning of every operation cycle may be unsatisfactory if the adaptation process starts afresh every cycle. The problem becomes more severe for robots with relatively short operation cycle as there is little time to complete its intended task. As most industrial robots are required to perform prescribed patterns of operation in the factory which remain unchanged from cycle to cycle, robust-adaptive control is recently receiving increasing attention. Robust adaptive control makes the iterative operations of a robot manipulator assume a closed-loop character in the sense that an attempt is made to reduce the error between the desired and actual motion in the current cycle by referring to the corresponding errors in the previous cycle(s). With proper control, a robotic system can accomplish any repetitive task precisely after robustly-adapting for a number of cycles. Before we address the Robust Adaptive Control (RAC) of robot manipulators, a brief overview of some other control methods for robotic systems are provided.

Figure 1-1 Standard robot manipulators [1].

1.2 Background and Motivation

After discussing about the robot manipulators [18]–[20], let us now discuss about the control of robot manipulators. Automatic control systems were first developed over two thousand years ago. The primary motivation for feedback control in times of antiquity was the need for the accurate determination of time. Around 270 B.C., the Greek Ktesibios invented a float regulator for a water clock, which kept time by regulating the water level in a vessel [21]. In the following we will only give a brief historical summary of works related to the control problem of robot manipulators. It is not intended to cite all the important contributors in this field. In 1848 J.C. Maxwell explained the instabilities exhibited by the fly ball governor using differential equations to describe the control system. The first servoed electrically powered teleoperator reported was developed in

1949 at Argonne National Laboratory (ANL). The teleoperator was manually controlled by a master-slave system and can be considered as an embryonic form of the present-day robotic manipulator. Unimation developed the first industrial robot in 1959 and the first Unimate robot was installed in a General Motors plant in 1961. The first Unimate robot operated with a simple controller in the robot itself, unlike the master-slave teleoperator. Although the first industrial robot was built by Unimation in 1959, most research in the control area was not published until the late 1960's.

The most common approach used in early industrial robots employed an independent Proportional-Derivative (PD) or Proportional-Integral-Derivative (PID) controller for each joint. This conventional joint servo control was investigated by Paul [1972] and Roderick [1976], and conditions which guarantee global stability for the method were presented by Takegaki and Arimoto [1981]. The independent joint control is the simplest and most widely used method today, the effects of which on robot manipulator performance were analyzed by Luh, Fisher and Paul [1981]. The independent joint control is usually considered as a linear control law, where each control input is a linear function of the output measurements. A form of quasi-linear multivariable control particularly suitable for teleoperator applications was the resolved motion rate control, presented by Whitney [1969]. Luh, Walker and Paul [1980] developed a pseudo-linear open-loop feedback law termed the resolved acceleration control which enjoys desirable dynamic stability properties in the absence of measurement and parameter errors. Another pseudo-linear feedback law with nonlinear pre- and post-processing of measurement and control signals was developed by Raibert and Craig [1981] for manipulator applications.

Since that time, control theory has made significant strides. Earlier control theory related to linear systems were very successful, therefore it was used in many industrial applications. Powerful methods used in this control theory made it highly useful in many control applications, but this control scheme was lately found to be insufficient, the reasons behind this control theory downfall was large operating range and precise performance requirement, which almost made linearized models invalid. Kahn and Roth [1971] applied the classical methods of optimal (open-loop) control to achieve approximately minimum-time point-to-point motion with both the path and speed unconstrained. Minimum-time motions, where the path was specified but the speed along the path was unconstrained, were considered by Bobrow, Dubowsky, and Gibson [1983]. A well accepted alternative approach for the dynamic control of robots is the so-called computed torque method, first proposed by Markewicz [1973] and Bejczy [1974], in which the equations of the robotic dynamics are used to evaluate the actuator torques necessary to produce the specified joint trajectory. Khatib et al [1978], and Liegois et al. [1980] are other early workers in this area [22].

Many systems in practical are nonlinear in nature and contains nonlinearities such as dead zones, coulomb friction, hysteresis, saturation and backlash. And these nonlinearities in general are referred to as “hard nonlinearities”, as these hard nonlinearities are discontinuous in nature and also linear approximations are not possible for them. Control systems with this nonlinearities causes unacceptable behavior (instability and limit cycles) within the whole system if they are not managed properly. In order to obtain desirable performance we need to use nonlinear control schemes and the design of these nonlinear controllers becomes complicated and very complex if the dynamics of the model is not known. With the complete knowledge of the system the design of these controllers becomes less complicated, and this can be seen taking an example of robot control where designing an stabilizing linear controller is far more complicated then designing a nonlinear controller for that system. One more thing that made implementation of nonlinear controllers much easier and cost effective were the low-cost microprocessors, with the use of these microprocessors nonlinear control became more popular and this field began growing very quickly during the past thirty years or so. Previously the performance of nonlinear control [23], [24] required complete knowledge about the system dynamics. But in reality, the dynamics of the system model which is calculated by applying physical laws and evaluating the required nonlinear functions from it may not be known in advance.

The parameters of the nonlinear function such as inertial mass which changes when robot grasp a new object, depends on the operating conditions and this inertial mass is also not precisely known in advance, thus effecting the performance of the system. Moreover with the phenomenon of aging effect, the parameters may be slowly time-varying and this type of uncertainties are called as parametric uncertainties [25]. Because of this parametric uncertainties present inside the system, it may cause the designed controller to degrade its performance and thus causing instability inside the system. Also, in almost all the mechanical systems there has inherent uncertain nonlinearities which results when the external disturbances and backlash as well as nonlinear friction force cannot be modelled exactly. This type of uncertain nonlinearities are generally classified as “unknown nonlinear functions”.

In order to increase the efficiency of the production lines, robot manipulators are mostly operated at high speed. During high speed robot operations issues of the nonlinear dynamics are even of more concern than in low speeds operations. At high operational speeds, the centrifugal, Coriolis and inertial forces are increased. Therefore, these effects should be addressed and eliminated in the dynamic model of the manipulator. Moreover, elasticity of the manipulator links, coupling effects between links, and gravity forces are effects that are often neglected. These effects are quite complex to determine and are called "unmodeled dynamics". But in case of operations demanding a high accuracy of the manipulator functions, these effects need to be considered in the dynamic model.

Finally from the above discussion it can be clearly seen that any systems in practical are subjected to both parametric uncertainties, unknown nonlinear functions and unmodeled dynamics. Therefore it becomes must to control these uncertain nonlinear dynamics of the system for making them usefully in any application. During the past fifteen years or so, there is a boost in research to minimize these uncertainties and control the dynamics of the system and enhance the precision, accuracy and performance considerably. Numerous algorithms were proposed for this purpose and the most common among them is adaptive control. The model accuracy and complexity increases with the number of the nonlinear effects included in the dynamic model. In order to avoid high complexity of the dynamic model some of the above mentioned effects are often omitted. Due to the fact that unmodeled dynamics are always present, use of modern control theory like adaptive control is necessary for the control of robot manipulator [26]–[28]. That is a difficult task that requires years of engineering expertise.

The problem of controlling robot manipulators can be divided into regulation and trajectory following problems. In this thesis we will investigate the trajectory following problem. As a nonlinear system a robot manipulator is characterized with several issues which have to be overcome in order to achieve control with satisfactory performance. The starting point for the successful design of the adaptive controller is an adequate dynamic model of the plant (in this case the robot manipulator). Obtaining an adequate dynamic model of the robot manipulator will provide the controller with information necessary to construct an appropriate control signal. The control signal will compensate for the nonlinearities.

The dynamic model of a robot manipulator has time varying parameters. When the manipulator grasps a load, the parameters of the dynamic model are changed. The dynamic model, which is an adequate representation of the system before the load seized, is not valid anymore. Adaptive control techniques have proven to be very successful in controlling systems with time varying parameters. Therefore, they are often used for control of robot manipulators. As a result, adaptive control has an ability to estimate the parameters true values and consequently generate an appropriate control signal. On the other hand conventional controllers do not have ability to adapt their control signal according to the changes in robot parameters. Accordingly, performance achieved with conventional controllers is degraded in comparison to adaptive controllers.

The literature on control of robot manipulators can be divided into several strategies. At first robot manipulators were controlled by fairly simple control algorithms. The computers available were extremely slow and expensive when compared to modern computers. For that reason implementation of the correct dynamic model of robot manipulator as a part of control algorithm was impossible. Accordingly, performance achieved with these control algorithms is just acceptable for the low operational speeds of the robot manipulators [29]. This class of control algorithms were based on linear control theory. The nonlinear robot manipulator dynamics in linear control are substituted with a linear approximation, in that approximation each joint of the robot manipulator is presented as a linear second order system. Moreover, coupling effects between the robot manipulator links were also omitted. The next step followed with the design of appropriate control based on the system applications, for example designing of an appropriate model reference adaptive controller (MRAC) or self-tuning regulator (STR) for that linear system. The main characteristic of these design procedure is that they lack proof of stability. This is due to the fact that linear control theory used in the design of these controllers is valid just for the systems with time invariant parameters [30]. As mentioned earlier, the robot manipulator is a system with time varying parameters. Because the influence of unmodeled dynamics is particularly strong at high operational speeds, these algorithms are only used for low speed robot applications.

The new group of algorithms is based on the theory of feedback linearization [31]–[33]. This is a powerful tool for the design nonlinear control systems. Feedback linearization transforms the nonlinear system into linear system. One can then apply linear control theory to the transformed system. It is important to distinguish between feedback linearization and linearization by Taylor’s series expansion. Feedback linearization is based on the principle of cancellation of nonlinear dynamics with a nonlinear control signal. The nonlinear control signal is actually output from the exact inverse of robot manipulator dynamic model, while the input is the desired robot trajectory. One of the main representatives of this approach is an algorithm introduced by Craig [30] also known as the computed torque method. This algorithm is proven to be globally stable. Moreover, it is asymptotically stable if the desired trajectory is persistently exciting. This algorithm requires the measurement of the position, velocity and acceleration signals of robot manipulators joints. That is a disadvantage because on most industrial robot manipulators there are no sensors for the velocity and acceleration signal [34]. On the other hand, if they are available they are often very noisy. Nevertheless, this algorithm requires computation of the inverse of the inertial matrix in the adaptation law and this can lead to algorithm failure in a case of singular inertial matrix.

As a result, researchers have investigate a new class of adaptive control algorithm that do not require the acceleration signal. Slotine and Li [35] introduced an adaptive control algorithm for robot manipulators that uses semi-positive definite Lyapunov function and requires persistent excitation to ensure parameter convergence. Otherwise, the algorithm could become unstable due to parameter drift [36]. The Slotine and Li algorithms does not require the acceleration measurement or the inversion of the inertial matrix. This is a major contribution of this algorithm over Craig's [30] algorithm. On the other hand, the main drawback of Slotine and Li's algorithm [30] are high sensitivity to noise, and dependence of error dynamics on the estimated parameters [36]. Schwartz [29] proposed an algorithm which goes one step further in terms of required signals. The Schwartz algorithm requires only the position signal to be measured, while the velocity signal is substituted by a velocity signal from the reference model which is always available and is not corrupted by noise. Stability of this algorithm is shown through computer simulations. We will make use of this technique later to design proposed adaptive control.

The new class of proposed algorithms requiring an observer is motivated by the fact that acceleration and velocity measurement are very noisy. As a result acceleration and velocity signals are estimated while the position signal is measured. These algorithm use the separation principle which allows the independent design of the observer and the state feedback controllers. The separation principle is a well-known technique for linear systems theory and is applicable to a special class of nonlinear systems [37], while for other nonlinear systems stability is not guaranteed if the measured states are substituted with their estimates [38]. Accordingly, Schwartz [39] proposed the MRAC algorithm which uses a linear observer to estimate the joint velocity signal instead of a nonlinear observer. This simplifies the implementation. The theoretical proof of stability for this algorithm does not exists. Lee and Khalil [40] have presented an output feedback adaptive algorithm which uses a high gain observer to estimate velocity signals. The algorithm uses as adaptive law with a parameter projection feature. The authors states that the algorithm recovers performance achieved under full state feedback. The algorithms have two main drawbacks; there are difficulties in implementation of high gain observers, and the algorithm requires estimation of several parameters before one can begin with controller design.

The proposed algorithm in this thesis is similar to Lee and Khalil [40] algorithm along with Schwartz [29] algorithm to control the robot manipulators. We use computer simulations to combine these two algorithms based on both performance and implementation. Simulations are performed with sinusoidal reference signals. Then the algorithms are tested with uncertainties and disturbances added to the system. Motivated with all the advancement in the control strategies of the robot manipulators, they are used in various industrial and non-industrial applications. Of all the robots in the world today, around 90% are used in industries and these robots are referred to as industrial robots. Out of these 90%, more than 50% are deployed in automobile industries. These robots also execute certain special tasks such as handling dangerous materials, assembling products, material handling, loading and unloading, continuous arc and spot welding, spray finishing, inspection, repetitive, backbreaking and unrewarding tasks, and task involving dangers to humans.

The advances in robotic technology are directed not merely to be used in industries. Parallel, robotic technology in non- industrial environments is also becoming popular. Robots are fast-finding their ways into research laboratories, energy plants, agriculture, hospitals, space, homes, textiles, services, education etc. The applications of robots are only limited by need and imagination of the developer and the end user. When purposefully employed, robots have endless potential to bring about drastic improvements in the economy, life style and overall quality of human lives [41].

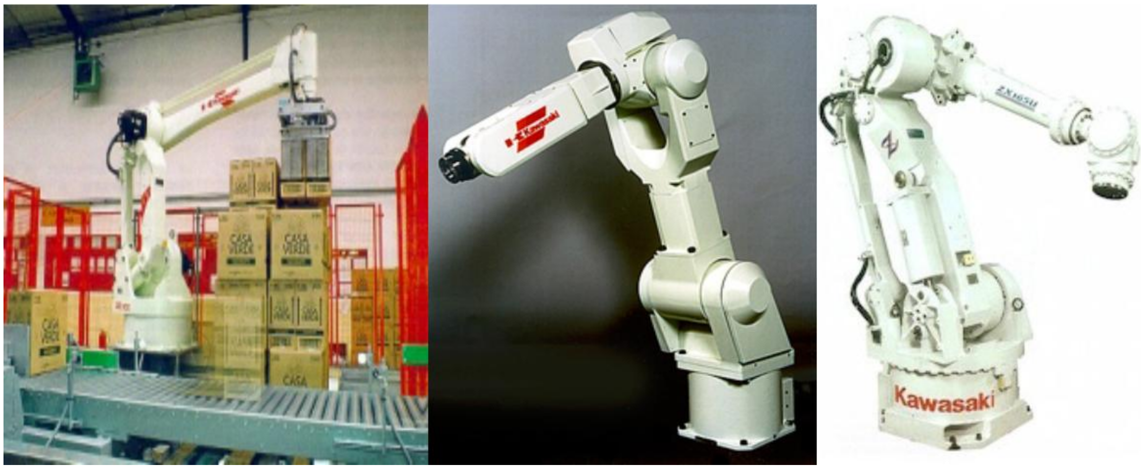


Figure 1-2 Shows the of robot manipulators in industrial applications such as material handling, assembling etc [41].

Figure 1-3 Implementing robot manipulators in space [41].

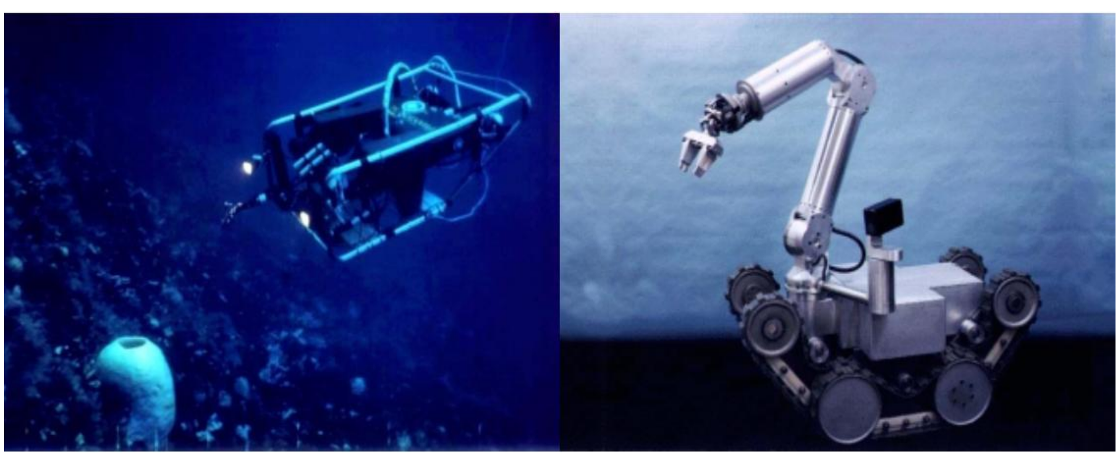


Figure 1-4 Operation of robot manipulators in hazardous environment, one is operating under water and one operating in atmospheres containing combustible gases [41].

Figure 1-5 Implementation in medical fields as robotic assistant for micro surgery [41].

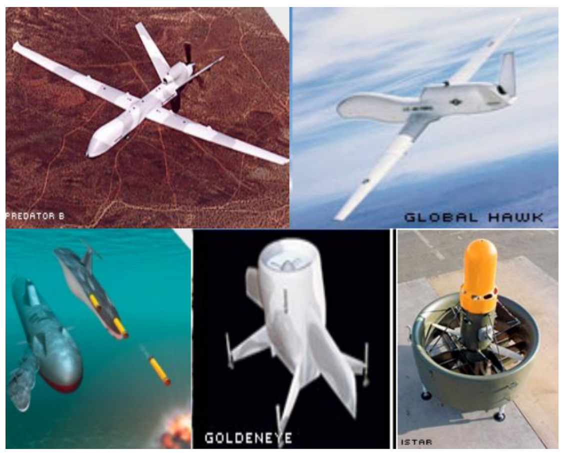


Figure 1-6 Implementation in military purposes [41].

In recent days DARPA (Defense Advanced Research Projects Agency) which is responsible for the development of emerging technologies for the use by the military started ARM (Autonomous Robotic Manipulation) program. ''The autonomous robotic manipulation program is creating manipulators with a high degree of autonomy capable of serving multiple military purposes across a wide variety of applications domains. Current robotic manipulation systems save lives and reduce casualties, but are limited when adapting to multiple environment and need burdensome human interaction and lengthy time durations for completing tasks. ARM seeks to enable autonomous manipulation systems to surpass the performance level of remote manipulation systems that are controlled directly by a human operator. The program will attempt to reach this goal by developing software and hardware that enables robots to autonomously grasp and manipulate objects in unstructured environments, with humans providing only high-level direction''. Some of the achievements of DARPA can be seen below [42].

Figure 1-7 Atlas- The Agile Anthropomorphic Robot (AAR) [42].

Atlas is a high performance, human like robot designed to work outdoor, rough terrain. It can walk like humans, carry weights and manipulate the surroundings in extremely rough terrain. It has 28 hydraulically-actuated degrees of freedom.

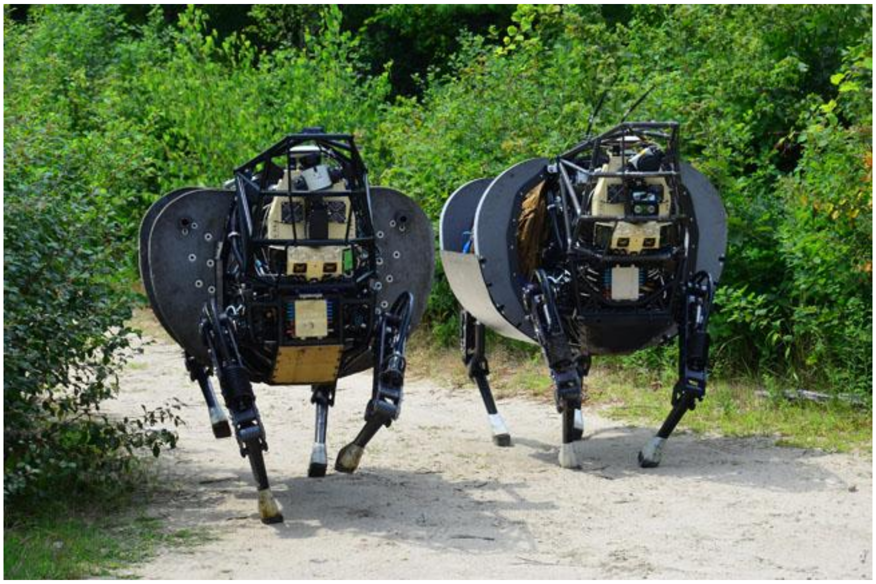


Figure 1-8 LS3-Legged Squad Support Systems [42].

LS3 is a rough-terrain robot designed to go anywhere Marines and Soldiers go on foot, helping carry their load. Each LS3 carries up to 400lbs of gear and enough fuel for a 20-mile mission lasting 24 hours.

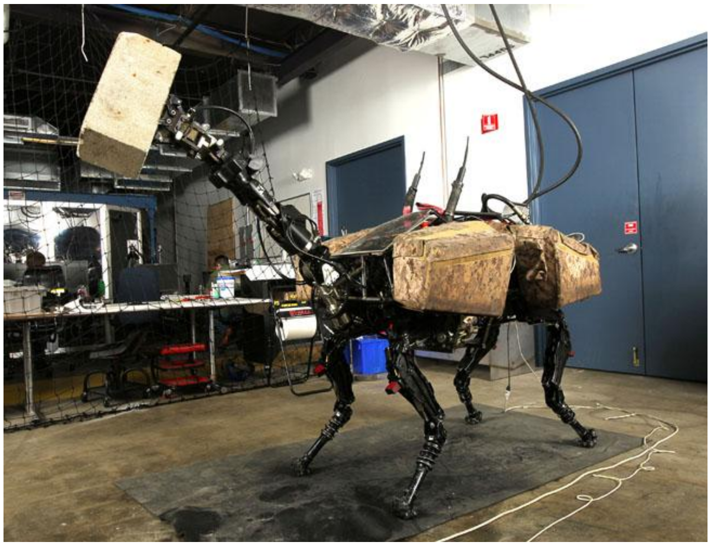


Figure 1-9 BigDog- The most advanced rough-terrain robot on earth [42].

BigDog is a rough-terrain robot that walks, runs, climbs and carries heavy loads. BigDog is powered by an engine that drives a hydraulic actuation system. BigDog is the size of a large dog or a mule; about 3 feet long. 2.5 feet tall and weighs 240lbs.

Figure 1-10 RHex- Devours Rough Terrain [42].

RHex is a six-legged robot with inherently high mobility. Powerful, independently controlled legs produce specialized gaits that devour rough terrain with minimal operator input. It has a complete sealed body, making it completely operational in any weather condition.

From all the applications discussed above it is clear that without proper communication between these robot manipulators, it is impossible for us to achieve such a high performance applications. Therefore, the proper channel of communication or also called as synchronization is the key point of any robotic application to attain high performance. Synchronization is a word, coined from two Greek words chronous (meaning time) and sign (meaning common) to imply occurring at the same time.

Synchronization effect in physical systems was first observed by Christian Huygens who found that two clocks supported from a common wooden support had the same rhythmic motion. Even when the clocks were disturbed they reestablished their rhythms. Later this phenomenon was found and investigated in different man made devices like electronic generators, musical instruments etc. Nature employs synchronization at different levels in biological systems. Synchronous variation of nuclei, synchronous firing of neurons, adjustment of heart rate with respiration, synchronous flashing of fire flies etc. are some such examples of natural phenomenon. Recently synchronization has found new meaning and relevance in the context of nonlinear systems that are capable of exhibiting chaotic or complex behavior. Such systems when coupled are found to reach synchronization [43]–[47]. The concept of synchronization will be clearly explained in the next chapters.

Our interest in this work will be mostly centered on synchronization of multi-robot manipulators using observer based robust adaptive control scheme in the presence of model uncertainties, unmodeled dynamics, external force disturbances etc.

1.3 Thesis Objectives

The primary objective of this thesis will be designing a high gain constant as well as time-varying observer based robust adaptive control for an uncertain robot manipulator in the presence of parametric uncertainties, non-parametric uncertainties, unmodeled dynamics, nonlinearities and external disturbances. To achieve this goal we carry out simulation on a two Degree Of Freedom (DOF) robot manipulator and make sure that tracking error convergence is achieved, so that the proposed design validates. After successfully achieving the primary goal, the secondary objective of the work will be comparing the results of constant high gain observer based control with time varying high gain observer based control law. Lastly, after the control law successfully validates the above objectives, then this proposed technique is implemented on multiple robot manipulators working in synchronous environment. In order to verify this objective, we carry out simulation on three identical two DOF robot manipulator and make sure of synchronization error convergence is achieved and conclude the thesis.

1.4 Thesis Organization

In Chapter 2, we outline the technological advancements in robotics, various control techniques starting from basic Proportional-Differential (PD) control to the most advanced robust adaptive control techniques are discussed. The need to design an observer based control is also high lightened in this chapter along with the basics of the synchronization concept of multiple robot manipulators.

In chapter 3, we discuss the dynamics of the robot manipulator, the most general model of the robot manipulators (Form-I and Form-II) are outlined and properties of the inertial matrix and matrix containing Coriolis, centripetal and gravitational forces are discussed.

In chapter 4, we present the general introduction of the observers along with its properties. Brief idea on the design procedure of linear are nonlinear systems observer is mentioned, followed by the observer-based control techniques. Then observer-based state feedback design is outlined with the high gain observers which is used in our work.

In Chapter 5, problem formulation for an uncertain robot manipulator is done, then the proposed robust adaptive controller is designed and analysis on its stability is done. After successfully deriving the control law observer design is implemented, to check the performance of the proposed control algorithm in tackling uncertainties and disturbances simulation results are examined related to both the constant and time varying high gain observer based control technique. The results are then compared and concluded.

In Chapter 6, the concept of synchronization is discussed based on multiple robot manipulators. We then derive the synchronization control algorithm based on robust adaptive control law. In order to examine the convergence of the tracking and synchronization error and boundedness of the closed-loop signals, simulations results are shown.

In Chapter 7, overall conclusion of the proposed work is presented.

CHAPTER 2

LITERATURE SURVEY

2.1 Introduction

Programmable, multifunctional robot manipulator have become mandatory, not only in industries but also fields like medicine, space, satellites and other hostile environments over last 40 years or so. Robot technology is advancing rapidly and has moved from theory to applications and from research laboratories to industries [48]. This growth of the robots has been taking rapid strides since the introduction of the robot manipulators in the industries in early 1960s. Still, achieving super humanoid anthropomorphic robot depicted in fiction seems to be a long way. This growth was made possible only because of the advancement of research in the field of robotics. In this chapter, we will see in brief about technological advancements in robotics, control of robot manipulators, need to design an observer and lastly synchronization phenomenon of multiple manipulator systems will be discussed [49].

2.2 Technological Advancement in Robotics

Technological developments have enabled engineers and designers to develop varieties of robots to suit different requirements. With the rapid growth in technology, the industries are fast moving from the current status of automation into robotization, to increase productivity and to deliver uniform quality. This requirement in turn has escalated the demand of robot technologies. Based on the requirements and characteristic breakthroughs in the robot capabilities, the growth of the robotic technology is grouped into four generations namely first, second, third and fourth generations [50].

The first generation robots are the repeating, non - servo, pick and place, or point to point kind. Nearly 80% of industries use these robots. It has been predicted that these first generation robots will continue to be in use for a long time. The addition of sensing devices and enabling the robot to alter its movement in response to sensory feedback marked the beginning of second generation. These robots exhibit path–control capabilities. This technological breakthrough came around 1980s and is yet to mature. The third generation robots are those enabled with human intelligence. The technological growth in computers led to high speed processing information and, thus, robots also acquired artificial intelligence, self-learning and conclusion – drawing capabilities through past experience [51]. On-line computation and control, artificial vision, and active force/ torque interaction with environments are some of the significant characteristics of these robots. This technology is still in its infancy and has to go a long way. Fourth generation may be a reality only during this millennium. It is difficult to predict its feature. It may be true android or artificial biological robot or a super humanoid capable of producing own clones. This might provide for fifth and higher generation robots [52].

A robot manipulator is required to carry out specific tasks by moving its end effector accurately and repeatedly. The execution of the specific task requires the manipulator to follow a preplanned path, which is largely a problem of trajectory planning and motion control for the manipulator [53]. The control systems play vital role in positioning the manipulator in the preplanned path. The efficiency of the control systems is based on the exact measurement of the parameter variations or errors. When the manipulator executes the task in free space and or the application of specific constant force, it is obvious that the internal forces like inertial, Coriolis, and frictional forces exerted by the environment also affect the performance of the robot. Due to the influence of these forces, there is a variation in the preplanned path of manipulator. These variations are called errors. The input torque applied is not sufficient to position the robot arm in the preplanned path. The insufficient torque affects the parameter of each links and joints, which means that there is a fluctuation in the joint positions and velocities of each joint. In control system, at every instant of time, the actual joint positions and velocities are measured by sensors such as encoders, and tachometers that are mounted on joints. These are used to compute the error between the desired and actual positions and velocities [54]. Even though errors can be measured, still there is a lack in measurements and quantification. This is due to the selection of dynamic model of the robot manipulator, selection of sensors, encoders and etc. Therefore, before moving to these fluctuations and let us see some control schemes used in as early stages of control.

2.3 Control of Robot Manipulators

Many factors contribute to the overall tracking performance of robot manipulators, and it is good engineering practice to take them into account in the system design. In terms of control, in particular, there are many control schemes and methodologies that can be applied. The particular control technique chosen can have a remarkable impact on the accomplishment of the manipulator and accordingly on the scope of its possible applications. For example, introducing the adaptive control technique can lead to precise tracking when the operation of a robot is varied. The control strategies which are commonly used, not specifically for varied or repetitive operations, are briefly reviewed in order to highlight the problems and their basic differences.

In initial stages of control Raibert and Craig [55], attempted dealing with the problem of rigid manipulator by applying hybrid position control scheme. Qu [56], analyzed the control of these nonlinear class of robot manipulator systems based on linear and coupled characteristics nature and found out that the dynamic of robot manipulator becomes much more complex. Matsuno and Yammamoto [57], they attempted to control the two link flexible manipulator system by applying the dynamic hybrid force control scheme. Krishnan and Mcclamroch [58], developed the nonlinear differential algebraic control systems for constrained robot systems. Steinbach [59], focused on optimizing error by boundary value problem approach. Later Lim and Seraji [60], discussed all the control strategies available at that time briefly, and came up with a suitable control scheme for the manipulator system.

In 1999, Shi et al [61], Analyzed and derived the mathematical model of a constrained rigid-flexible manipulator based on Hamilton's principle. Oucheriah [62] and Ho et al [63], discussed the error minimization in the delayed system and concentrated on the constraints functions for reducing the inaccuracy levels. Featherstone and Orin [64], Ata and Ghazy [65] concentrated on developing an algorithm for dynamic computations and focused on dynamic modelling and simulation of constrained motion of rigid manipulator in contact with a compliant surface. Bianco and Piazzi [66], applied the global optimization approach to obtain the minimum time by considering the joint torque and derivatives for nonlinear manipulator dynamics. Ata et al [65], attempted to find an optimal motion trajectory for the constrained motion based on minimum energy consumption. Hopler et al [7], addressed the equations of motion and sensitivity for optimal control problem in legged robot. Kalyoncu and Botsal [67], Chitta et al analyzed the elastic manipulator under time-varying cases and addressed the dynamic equations of motion. Mist et al [62], discussed the performance of the robot on the bases of finding the positions of base and joint angle of 6 degree of freedom manipulator.

Mostly, many of the control schemes mentioned in the early stages of control did not account the uncertainties, unmodeled dynamics and external force disturbances effect in the dynamics of the robot manipulator, which made it limited to certain applications. These uncertainties and the disturbances are linked to the system and cannot be neglected, they can only be minimized effectively so as to get the required work done by the manipulators. Therefore, in the later stages of control researches were focused on various control strategies that focused on the minimization of these unwanted disturbances and increase the system accuracy, precision and repeatability. Some the control schemes used in the later stages for control are discussed below.

In one of the technique used is to linearize the robotic system about the nominal desired trajectory, and then various schemes of linear system are use [68]. A major problem with this approach was that linear control laws are applicable only in the neighborhood of the given nominal trajectories. It’s not applicable to the complete system as a whole. In order to overcome the above problem of linearization at a nominal trajectory, [69] uses gain scheduling algorithms which linearizes the system about number of nominal trajectories to reduce the hindrance. This method splits the system into sub-systems and then linearized around nominal trajectory. In [70] a nonlinear transformation is developed which converted the given nonlinear system into controllable canonical form. Converting to this forms is equivalent to linear form, and from here linear control schemes can be used. The development of this transformation is nontrivial, which is the major drawback of this method.

So, it was now understood that linearization approaches were insufficient for the control of these manipulators. Independent-joint PD control was by far the most popular linear feedback control method for contemporary industrial robots, one of feature of PD control was that it introduced heavy damping during the fastest parts of the movement, where it is not particularly needed. The main shortcoming of this form of controller is that, although it may produce an asymptotically stable system, the transient response may not be acceptable. The joint motions may show too much overshoot and oscillation. The oscillatory behavior can be avoided through the use of large damping coefficients, but this may make the speed of response much slower than permitted by the capabilities of the actuators. Moreover, the coupled property of the joint dynamics makes it difficult to determine appropriate feedback gains which are usually selected by independently tuning each joint.

Ambrocio Loredo-Flares [71] discussed the possibility of applying neural networks to vision-based robot control. Christian Eitner [72] applied the neural networks to trajectory control of the robot manipulator. The weights of the neural networks were adjusted in real-time such as to minimize the error between the desired and current robot states in order to cope with unknown and time-varying dynamics. Jincheng Yu and Guanghan Xie [73] proposed a neural network controller for the force control of robots manipulators handling unknown objects. There have appeared many other studies using neural networks for robotic control [73], [74]. However, it seems that many of the neural-network-based control as reported so far does not fully use a priori information of the dynamic system, which can be obtained by utilizing physical laws governing the robot manipulator behavior. Because knowledge about the formulation of the rigid-body dynamic system is normally ignored in the neural network strategy, the method fails to provide perfect tracking control.

The Variable Structure Control (VSC) was then developed and it had several interesting and important properties that cannot be easily obtained by other approaches. Russian authors initially proposed the Variable Structure (VS) theory in the 60's using the mathematical work of Filippov [75]. Several researchers have been studying it, both in continuous [76], and in discrete time [77]. VS controllers provide an effective, robust way for controlling nonlinear plants and its roots are the bang-bang and relay control theory [78]. The term VS control arose because the controller's structure is intentionally changed according to some rules in order to obtain the desired plant behavior. Due to this change in structure, the resulting control law is nonlinear. Nevertheless, this technique was seen to be inappropriate for robot manipulators used for discontinuous operations. To avoid this sliding mode control scheme was developed by Esfandiari [79], the limitation with the use of sliding mode is the high frequency switching, commonly known as chattering. Chattering is unacceptable in robotics as it may excite unmodeled high frequency modes, which could damage the robot manipulator and high frequency plant dynamics [80]. The general approach undertaken to overcome this problem is to replace the non-linear switching function by a smooth one as in Slotine and Li [32] and

Ambrosino et al. [81]. This method, however, seriously alters the performance of the controller, because of the high degree of smoothness needed to completely overcome chattering.

The approaches used to control robot manipulators such as linearization control, variable structure control, sliding mode control were insufficient and in effective because each control scheme had its own drawback, now it was time for the researchers to find out control schemes which were more flexible and reliable to the uncertainties and disturbances caused to the system. Moreover, the dynamics of the robot manipulator are highly nonlinear, as a result of that we are unable to design the controller because we don't know the system parameters. Even if the system parameters are known, the use of the linear controller or variable structure control with constant parameters could result in an unstable closed-loop system because of controller's inability to adjust its output to the changing system. Accordingly, development of a control strategy that has the ability to successfully solve that problem was necessary. That control strategy is called adaptive control.

Adaptive controllers have parameters that are changing along with change of the system parameters. When using adaptation or estimation techniques, the adaptive controllers is capable of increasing accuracy of the controlled system when the parameters of the system are unknown. The mechanism that gives the relation between varying parameters of the plant and adjustable controllers is called adaptive control law. The purpose of the estimator is to estimate unknown or time-varying parameters. The adaptive controller is nonlinear due to the fact that the adaptation laws is always nonlinear. Generally, an adaptive controller has two loops: the normal feedback loop and the parameter estimation loop. The normal feedback loop is the actually an ordinary controller. The parameter adjustment loop is an adaptation mechanism whose purpose is to adjust the controller parameters in order to decrease the error between the actual plant output and the desired output.

There are two main concepts of adaptive control: direct adaptive control and indirect adaptive control. In the concept of the direct adaptive control, the parameters of adaptive controllers are directly updated form the adaptation law. On the other hand, in indirect adaptive control, the plant parameters are estimated first and then the controller parameters are calculated by using the plant parameters estimates and algebraic design equations [82]. The main representation of the direct adaptive control concept is Model Reference Adaptive Control (MRAC). Generally, MRAC consist of four main parts such as the reference model, the controller, the adaption law and the plant. Nevertheless, we have reference trajectory instead of reference model, due to the fact that this thesis is focused on trajectory tracking control of robot manipulators and rest all remains the same. The general block diagram of MRAC is given in Figure. 2-1.

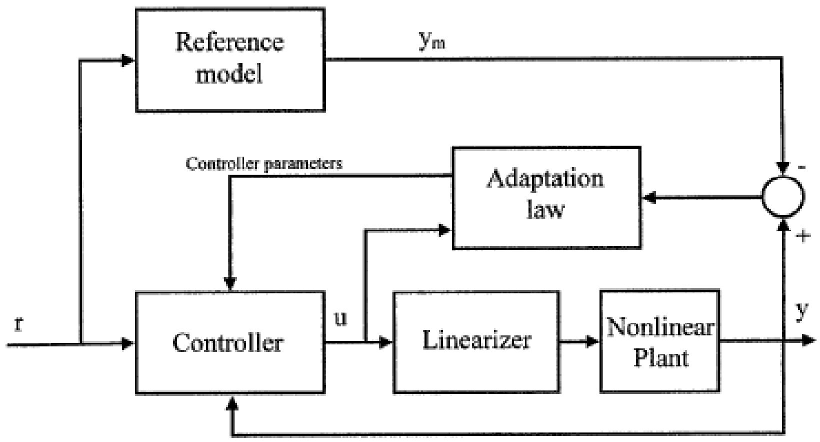


Figure 2-1 General block diagram of a model reference adaptive control [37].

The plant of the adaptive control system usually has unknown and time-varying parameters. Also, the system can have known and constant parameters. In that case, the ordinary feedback control will do the job. Moreover, in the case of a plant with known and constant parameters, the use of adaptive control is not recommended due to its complexity and higher implementation costs. In order to be able to apply the adaptive control technique, the structure of the dynamic model of the plant has to be known. The reference model, in our case reference trajectory defines the ideal behavior of the adaptive control system. The controller parameters will be updated as long as the difference between reference trajectory output and the actual plant output is different from zero. The reference trajectory is defined by the designer of the adaptive control system in a way that the response of the reference trajectory y_r to the reference signal r meets the designer specifications. The controller of the adaptive control system has to be parameterized, otherwise, it would not be possible to update its parameters when they are estimated. The controller has to be capable of achieving perfect tracking of the reference model output when the parameters of the plant are known, otherwise use of the adaptive control is pointless. The parameters of the controller will be updated until perfect tracking is achieved. At that moment the estimates of the plant parameters are equal to their true values, assuming that the reference signal is “rich” enough. The purpose of the adaptation law is to constantly vary the controller parameters until the tracking error is minimized to zero. In order to achieve that goal, the adaptation law needs the difference of the actual and desired output signal and the control signal u as input signal [83].

In the case of conventional controllers, the stability of the closed loop system can be proven in a number of ways using classical stability analysis techniques such as root-locus, Nyquist criterion, Hurwitz criterion and others. In adaptive control systems the stability of the closed loop systems is hard to prove particularly when the plant is nonlinear. In that case, the use modern stability theory such as Lyapunov and hyper-stability theory is necessary. The problem of proving the stability of the adaptive control systems is complex because the closed loop systems has to be stable even when the parameters of the plant are varying. Actually, the system has to be stable for infinite set of plants with the same structure and different parameters values.

The purpose of the linearizer is to compensate the nonlinear dynamics of the nonlinear plant. The use of the linearizer block results in the transformation of the nonlinear state equations into linear form, by cancelling original plant nonlinearities. Using the principles, of the feedback linearization, the robot manipulator as a nonlinear system with coupling effects between the joints can be transformed into linear system where each joint is treated as a double integrator. That combination of linearizer is not required in this thesis because we make use of the nonlinear model of the robot manipulator itself along with adaptive sub-controllers to control the dynamic model of the system [84]. Now moving to indirect adaptive control, the representation of the concept of indirect adaptive control is the self-tuning regulator [28]. The block diagram of the self-tuning regulator is illustrated in Figure. 2-2.

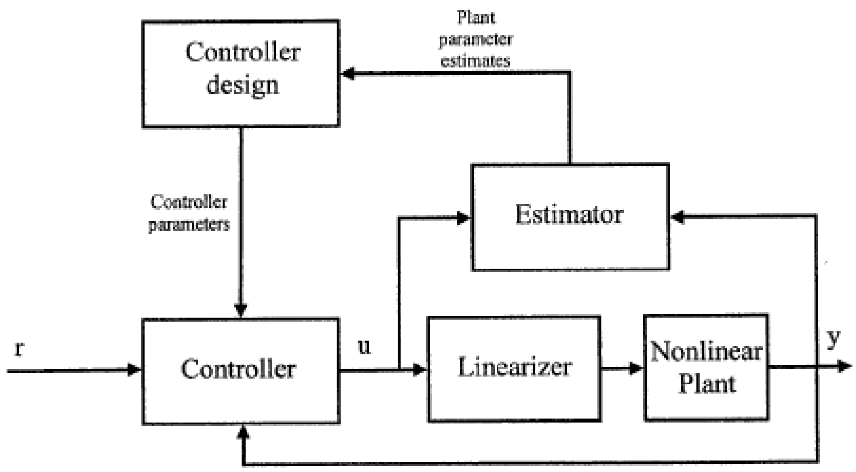


Figure 2-2 Block diagram of a self-tuning regulator [82].

The concept of indirect adaptive control has two loops like the concept of direct adaptive control. The first loop consists of the controller and the plant. Within the second loop, the plant parameters are estimated and then the controller parameters are calculated. The first loop is identical in both concepts of adaptive control. The controlled plant has unknown or varying parameters. At each sample time the estimator provides the controller design block with estimates of the plant parameters. By using estimates of the plant parameters as if they are true parameters values, the controller design block computes the controller parameters by solving an algebraic design equation. When obtained, the controller parameters are passed to the controller that is parameterized with adjustable parameters. Now, the controller will be able to generate control signals based on the reference feedback signals. This process is repeated at every sample of time.

A purpose of an estimator is to provide the estimates of the plant parameters. The estimator uses some of the parameter estimation techniques such as Recursive Least Square algorithm (RLS) and the plant input and output signal to produce the parameter estimates. If the input signal to the plant is “rich” enough, the convergence of the plant parameter estimates to their true value is faster in the case of indirect adaptive control than in the case of direct adaptive control. This is due to the fact that the direct adaptation techniques are gradient based while the indirect adaptation techniques are based on RLS algorithms [85]. The main disadvantage on indirect adaptive control is performance degradation that can lead to instability when the input signal to the plant is constant or equal to zero. The problem can be partially solved by addition of a perturbation signal to the input signal. Moreover, separation of the controller and estimator design has resulted in lack of proof of stability for indirect adaptive control algorithms. Due to these disadvantages, the direct adaptive control algorithms are preferred in applications where stability is an issue. The algorithms presented in this thesis are direct adaptive control algorithms.

Huy-Tung Le [86], proposed an adaptive asymptotic stable control scheme in order to compensate the effects of disturbance and friction and control the manipulator. By using coordinate transformation and state-space feedback the nonlinear model of the robot manipulator is transformed into linear one. The adaptive asymptotic scheme used here is integration of model reference technique combined with exact linearization. The main drawback for this approach is that uncertainty in parameter is not accounted and disturbance is bounded. C. Canudas De Wit [87], used a control scheme which was combination of the adaptive control law with sliding observer. The observer is based on sliding control design, which defines a switching surface and the dynamics of this surface is determined by Filippov's solution. The drawback for the proposed control is that sliding controller used in sliding observer generates "Chattering", chattering add up large amount of high frequency components thus making to unsuitable for control law. M. Danesh Sararoody [88], used a new scheme for adaptive control of manipulator to estimate and reject the external disturbances. One of the best adaptive control scheme was developed in [89] for the robot manipulator systems with uncertainties and unknown parameters, where the upper and lower bounds for the uncertainties are decided with prior knowledge of the input signal. The only drawback of this control design was that the control gain was very high at the beginning in order to neutralize the effect of uncertainties.

The adaptive approach presented in [90] makes two major assumptions - firstly, that the inverse of the estimated inertia matrix is always bounded, and secondly, that measurements of joint acceleration are available. To satisfy the first assumption, parameter estimates are restricted to lie within a-priori known bounds of the actual parameters [91]. Spong and Ortega [92] propose using fixed a-priori estimates of the dynamics and an additional outer loop control component which is chosen adaptively to compensate for the inaccurate dynamics. The same update law as [90] is used, and hence the measurement of joint accelerations is still required for parameter estimation. Middleton et al. [93] also build upon the Adaptive Computed Torque (ACT) controller [26] but eliminate the need for joint acceleration to perform the parameter estimates by filtering the linear parameterization equations such that the repressor matrix $\emptyset$ is a function strictly of joint angles and velocity. Other researchers address this issue through the use of nonlinear observers to perform state estimation [94] of the joint accelerations.

A number of adaptive laws have been proposed [95] based on the preservation of passivity properties of the Rigid Body Dynamics (RBD) model of robot manipulator. Although they differ from the class of controllers originating from [96], the motivation for these schemes is also to eliminate the need for joint acceleration measurement [25]. Despite the numerous advancements, the adaptive methods presented thus far are still reliant upon adequate knowledge of the structure of the dynamic model and are thus particularly susceptible to the effects of unmodeled dynamics. In an attempt to account for this, the dynamic equation of robot manipulator is used in adaptive control laws has been extended to include additional dynamic effects such as actuator dynamics [97] and joint friction [98]. However, in many cases, such as with friction [99], simplified dynamic models are often used to approximate physical processes which, in reality, are complex and highly nonlinear. Consequently, dealing with the effects of unmodeled dynamics remains an open research area within adaptive control, and some researchers have combined adaptive control with robust control techniques [100] and even learning control [101].

So far we have seen that adaptive controllers deal with model uncertainty by attempting to identify a more accurate model of the system through parameter update laws, these update laws changes with time and are not applicable for wide range of uncertainties. This particular drawback of this scheme can be eliminated by robust control schemes. Robust control schemes focus on the development of the control strategies which can satisfy a given performance criteria over a range of uncertainty. In the robust controllers, the controller has a fixed structure with known bounds of uncertainty and no learning behavior takes place. The robust controllers have attractive features compared to adaptive controllers, which are [102]

- Ability to deal with disturbances.

- Ability to handle quickly varying parameters and unmodeled dynamics.

- They are easy to implement.

These controllers can achieve desired transient response and also convergence of their tracking error is uniform and bounded [103]. The survey on robust control strategies [104] and [105] shows that these kind of controllers are well known and very useful for different applications.

Various strategies have been proposed for robust control. Anticipating that the inexact linearization and decoupling due to model uncertainty will introduce nonlinearities into the error dynamics, well-known multivariable non-linear control techniques such as the total stability theorem [106], Youla parameterization and H_∞ [107], are used to design compensators which guarantee convergence and stability of the system error for a given set of nonlinearities. However, the application of these non-linear multivariable techniques can often result in high-gain systems [108]. An alternative solution to dealing with model uncertainty is the application of variable structure controllers such as sliding mode control [109]. The main feature of such controllers is that the nonlinear dynamic behavior of the system is altered through the use of discontinuous control signals which drive the system dynamics to `slide' across a surface where the system can be approximated by an exponentially stable linear time invariant system. Hence, asymptotic stability of the tracking error can be achieved [110] even in the presence of model uncertainty. Despite this advantage, due to the discontinuous control signals sliding mode systems are susceptible to control chattering, which may result in the excitation of high-frequency dynamic behavior [111]. Therefore, these robust control techniques are used with adaptive control in order to get the best performance of the robot manipulator from it. In thesis we also make use of robust adaptive control schemes, in order to minimize the uncertainties and external disturbances and control the robot manipulator.

2.4 Need for Observer Design

This design robust adaptive control also deal with the flaw that, if some of the states that are necessary for implementation of the control and adaptation laws are unavailable or too noisy, the observer has to be added with the robust adaptive control mechanism. In practical applications that is always the case. For instance, robot manipulators usually have available just the joint position signals while the velocity and acceleration signals are not used because they are corrupted by noise. The observer uses the plant output signal y and control signal u to estimate unknown states. An observer can be viewed as an algorithm that can reconstruct the internal unmeasurable states of the system from the measurable output. It is seen in the case of linear systems the observer theory is well investigated and the observability and detectability properties are closely connected to the existence of observers with strong convergence properties. However, in the case of nonlinear systems, the observer design problems has a systematic solution only nonlinearities are functions of the measurable output and the input. There are different observers designed based on states of the systems such as reduced order observers, full order observers and functional observers. There has been a significant work done on the functional kind of observers, let us briefly literature review related to the observers.

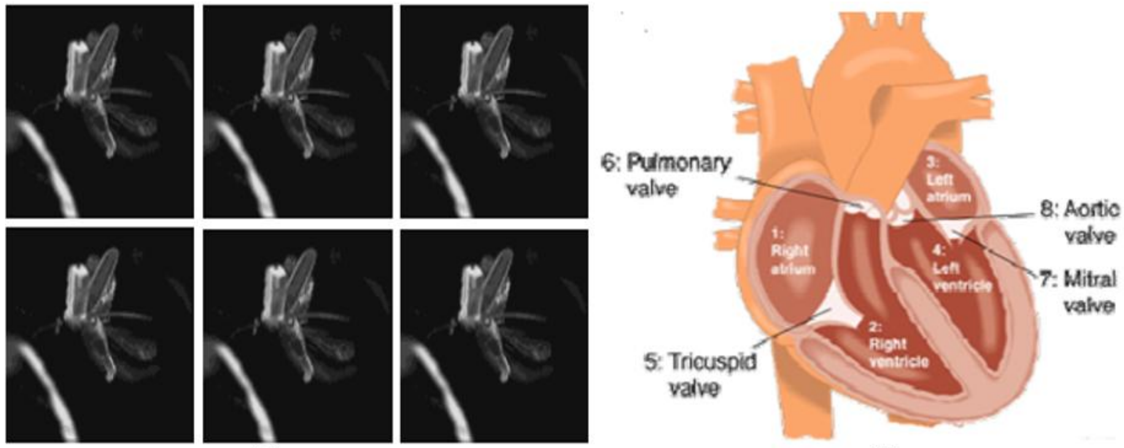
Zongli Lin [112], he discussed the problem related to robust semi-global or practical stabilization problem for Multi-Input and Multi-Output (MIMO) minimum phase linearizable systems, as a control he used output feedback along with squaring-up design [113]. Juhoon Back and Hyungbo Shim [114], used Linear Disturbance Observer (LDO) technique along with output-feedback controller to compensate the effect of disturbances in uncertain nonlinear system for example robot manipulator and this techniques was expected to be implemented in practical applications for increasing the performance of the closed loop system. Jung Rae Ryoo [115], explained the phenomenon of optical track following disc drive control systems using robust disturbance observer and feedback compensator, and simulation results were then implemented on two degree of freedom robot manipulator to show the effectiveness of the control algorithm. Many other observers were also designed based on feedback design, variable structure control, backstepping etc.

The observer used in this thesis is a nonlinear High Gain Observer (HGO), because of its ability to robustly estimate the unmeasured states and derivatives of the output, while asymptotically attenuating disturbances [116]. In general, HGO's have proved handy in nonlinear control, [117] gives us the complete idea about the development of HGO's over the past three decades. Finally, the control scheme is clearly defined, now we need to know the concept of synchronization in order to implement the proposed control strategy on multiple robot manipulator systems.

2.5 Synchronization of Multiple Robot Manipulators

Synchronization is a phenomenon that is widely encountered in nature, life sciences and engineering. There exist various synchronization definitions in various research fields. The general definition for synchronization is the adjustment of rhythms of oscillating systems due to their weak interaction. Synchronization problem depends on the type of applications that require suitable properties and comparison functions. Different applications require different properties and comparison functions. Throughout our study, we choose the comparison function to be the difference of the state's variables of the systems in hand.

Recently synchronization has found new meaning and relevance in the context of nonlinear systems that are capable of exhibiting chaotic or complex behavior [118]. Such systems when coupled are found to reach synchronization [119]. This finds wide application in coupled systems used in communication and image processing. Early observations of synchrony go back a few centuries to Huygens [120], who observed the phenomenon in weakly coupled pendulum clocks hanging from the same beam. Synchronization of oscillators is ubiquitous in nature. Examples include pacemaker cells in heart [121], the nervous system [122], glycolytic synchrony in yeast cell suspensions [123], flashing of fireflies [124], chirping of crickets in unison [125] and so on.



(a) (b)

Figure 2-3 Biological inspiration of synchronization control (a) Fireflies flashing unison, (b) Pacemaker cell [121].

In chaotic systems, owing to the property of sensitive dependence on initial conditions, it is not obvious that two chaotic systems can synchronize. A major discovery was made by Pecora and Caroll [126], who observed synchronization in two identical chaotic systems coupled in a "master-slave" setting. Considerable work has subsequently been done in studying synchronization in coupled nonlinear systems [127]. In recent years, experimental studies of synchronization in chaotic systems have been carried out in diverse areas such as laser systems [128], solid state physics [129], electronics [130], biology [131] and communication [130]. The synchronization scheme used in thesis is cooperative synchronization and various other control schemes are discussed in later chapters.

2.6 Summary

In this chapter, we have discussed the technological advancement of robot manipulators and outlined various control strategies used for the purpose to control robot manipulator. All the control techniques such as linear control, optimal control, fuzzy control, feedback control, variable structure control, adaptive control and robust control subject to uncertainties were examined, merits and demerits of each control was given in brief. Then we discuss about the robust adaptive control scheme used this thesis and mentioned its importance. Further, we mentioned the necessity to design an observer for the robot manipulator systems and discussed some observer designs along with high gain observer, which is used in our work. Lastly, we discussed about the synchronization of multiple robot manipulators.

CHAPTER 3

MODELLING OF ROBOT MANIPULATOR

3.1 Introduction

One of the most complex steps in designing a control system is deriving a mathematical model of the plant. This is an even more complex task in the case of a nonlinear plant such as robot manipulator. Nonlinear systems have much more complex dynamics than linear systems. Nonlinearities such as dead zones, backlashes and hysteresis can be minimized by precise construction of joint and gears. On the other hand, nonlinearities such as Coriolis and centripetal forces cannot be minimized by precise construction. Moreover, their influence increases with the square of operational speed. If they are not included in the dynamic model, accuracy of the robot control will degrade quickly with the increase of the robot's operational speed.

In practical engineering control problems, analysis starts with modelling of the robot manipulators or the physical system under study. The objective of the modelling is to establish the mathematical equations, model, as a set of analytical relations describing the dynamic behavior of the robot manipulators. The modelling process depends on the characteristics of the arm to be studied and the physical details to be included. This is why dynamic modelling, according to Gawthrop [132] and Brussel et al [133], incorporates several stages which can be summarized in:

1. Physical modelling.

2. Model simplification (schematic model).

3. Mathematical modelling.

4. Mathematical model analysis.

5. Model validation.

In the above stages the robot manipulators, or generally a dynamic systems would undergo several transformation and simplifications, for instance in the first stage an imaginary model of the robot arm is built essentially like the real system or from the design requirements of a robot arm [134]. At this stage many decisions are to be made concerning aspects such as friction, compliances of links and joints, linearity, noise, etc. A summary of the effects of some approximations on the mathematical model are shown in Table 1.

Table 1 Effects of approximations on the mathematical model.

Serial no.	Approximations	Mathematical model simplification
1	Neglecting small effects.	Reduces number and complexity of differential equations.
2	Assume environment independent of system motions.	Same as 1.
3	Replace distributed characteristics with appropriate lumped elements.	Leads to ordinary, rather than partial, differential equations.
4	Assume linear relationships.	Makes equations linear and allows superposition of solutions.
5	Assume constant parameters.	Leads to constant coefficient in differential equations.

Model simplification consists of establishing links connectivity and the nature of their relative motion in a schematic form to give more insight to help generate the right equations of motion. In the mathematical modelling process, the first point to consider is to select the state variables, which describe essentially, the storage of energy and mass in the system. The state variables of a robot manipulator are the positions and velocities of its links when the rigid mechanical system only is considered. Next step is the application of balance equation for force, moment, mass, energy or writing system elements relations which describe relative motion of links. Mathematical model analysis (Simulation) is the next step. The obtained equations of motion are used to imitate the behavior of the real system under a stimuli representing the action of a real control system or force/torque applied to the system [1].

Figure 3-1 ABB robot family and the IRC5 controller [1].

Figure 3-2 Three examples of robot structures from ABB. The parallel arm robot IRB340 (left), the parallel linkage robot IRB4400 (middle), and the elbow robot IRB4600 (right) [1].

The behavior analysis at this stage is useful for the of suitable control system and for the evaluation of the structure and parameters of the manipulator under consideration. Model validation is a necessary step at this stage. The obtained equations represent the dynamic model of the real system under study after several approximations. Therefore, it must be validated in order to obtain enough confidence that it adequately represents the robot manipulators dynamic behavior under a set of conditions determined by the purpose of the modelling. This means that the validation process involves comparison of the mathematical model solutions (simulation) with the real robot manipulators behavior subjected to the same stimuli. Usually, a model is not determined as absolutely valid, but rather, evaluation and model tuning are conducted until sufficient confidence is established within the context of intended uses of the robot manipulator.

3.2 Dynamics of Robot Manipulator

Dynamics of manipulators is a special case of dynamics of mechanisms, and is a field on which many books have been written. However, the work reported here is an attempt to analyze and use certain formulations of the dynamics problem which seem particularly well suited to application to manipulators. There are two major problems related to the dynamics of a manipulator that should be solved. In the first, a required trajectory is given in terms of $q_r, \dot{q}_r,$ and $\ddot{q}_r$ and the vector of joint torques, τ is to be found. This formulation is useful for the problem of controlling manipulators. The second problem is the opposite task to the first, which involves calculating how the mechanism will move under application of a set of joint torques. This is useful for manipulator simulation and some control schemes such as computed torque control [135].

3.2.1 Model of Robot Manipulator

The dynamic model of a robot gives a relation between torques that cause motion on one side of the equation and manipulator joint positions, velocities and accelerations on the other side. In order to obtain it, one will need to apply one of the methods of analytical mechanics such as Newton-Euler or the more efficient Euler-Lagrange method. The Euler-Lagrange method is the most common method used for derivation of the robot model. In order to use it one has to determine the Lagrangian of the plant. The Lagrangian is defined as the difference in the kinetic energy of the system and its potential energy. Accordingly, the first is to obtain equations for the velocity of each link. The potential energy can be calculated from the link position. Hence, the Lagrangian of the system can be determined. Applying Lagrange's equations to the robot manipulator will result in a general solution for robot motions given by (3.1) [136].

$$\tau = M(q)\ddot{q} + h(q, \dot{q}) - J^{T}(q)\, F_{ext}(t) \tag{3.1}$$

The term $\tau \epsilon R^{nx1}$ is the vector of torques applied on the manipulator, while n is the number of joints, $q \epsilon R^{nx1}$ is the position of the robot joint, $M \epsilon R^{nxn}$ is the manipulator mass matrix, $h \epsilon R^{nx1}$ is the vector of Coriolis, centrifugal and gravitational forces, $J \epsilon R^{nxn}$ is the task space Jacobian matrix with appropriate dimensions, $F_{ext} \epsilon R^{nx1}$ is the external force vector exerted on the end effector. The manipulator model given by (3.1) is considered to be very detailed model because most of the robot nonlinear dynamics are addressed in it.

The complexity of the model will result in complex analysis and computational inefficiency. Therefore some model simplifications are welcomed. Inertial torques represented by matrix M vary with manipulator motions as well with load changes, and cannot be omitted. Coriolis, centrifugal and gravitational forces represented by vector h cannot be omitted as well, because their influence increases drastically at the high operating speeds of the robot manipulator. High operating speeds are very common for modern manipulators. Moreover, if we consider that the robot manipulator operates in a horizontal plane, gravity forces can be neglected with significant error in the model representation [62]. In favor of that is the fact that the influence of gravity forces on manipulator links can be compensated by feedforward term [137]. In that case analysis conducted with ignored gravity forces is still conceptually valid.

Friction forces consist of two parts, viscous friction which is function of joint velocity and coulomb friction which is constant but sign dependent on joint velocity. Friction can be significantly reduced with the careful design of manipulator joints and use of high performance lubricants. As a result of that friction forces can be neglected in the dynamic model of the manipulator. Friction is a process that cannot accurately be modeled with a deterministic model. According to the control literature [138].

3.2.2 Model of Robot Manipulator-Form I

According to the recommendation given in the previous paragraph, the equation of the dynamics for a two degree of freedom (DOF) rigid body robot, which is illustrated in Figure. 3-3 and presented in joint space in Figure. 3-4, is given with the following equations,

$$\tau = M(q)\ddot{q} + C(q, \dot{q}) - J^T(q)\, F_{ext}(t) \tag{3.2}$$

The gravity term is neglected from h matrix given in (3.1), because the manipulator is operating in a horizontal plane. The manipulator has 2 joints, one between ground and the first link and the second one between the links. The links have an equal length $l_1 = l_2 = l = 1m$ and concentrated masses m_1 and m_2 at the end of each link as it is shown in Figure. 3-4.

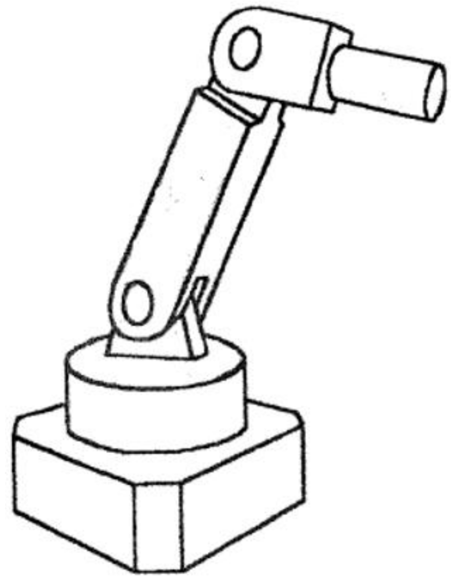


Figure 3-3 Two DOF Robot Manipulator [138].

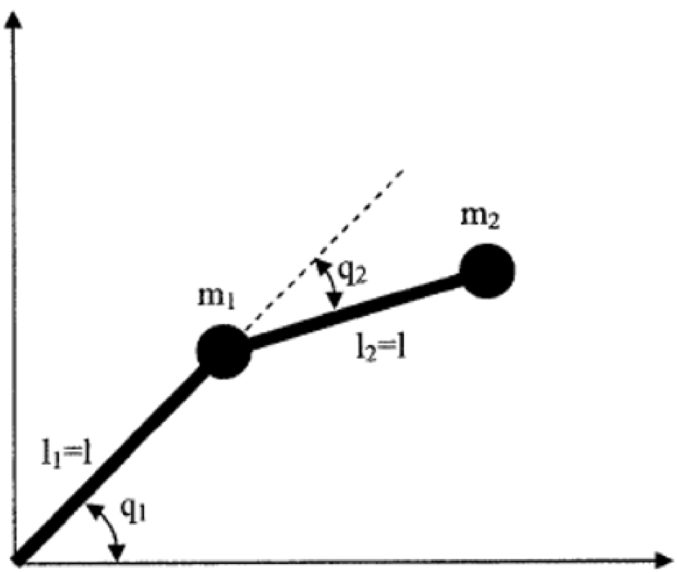


Figure 3-4 Simulated Robot in Joint Space [138].

According to Schwartz [36], the inertial matrix M and vector C representing Coriolis and centrifugal forces are given as:

$$M(q) = \begin{bmatrix} m_1\, {l_1}^2 + m_2\, \overline{l_2}^2 & m_2 l_2\,(l_2 + l_1 \cos\, q_2) \\ m_2 l_2\,(l_2 + l_1 \cos\, q_2) & m_2 {l_2}^2 \end{bmatrix} \tag{3.3}$$

$$h\,(\mathrm{q}\,,\dot{\mathrm{q}}\,) = m_2\, l_1\, l_2 \begin{bmatrix} -\,(2\,\dot{\mathrm{q}}_1\,\dot{\mathrm{q}}_2 + \dot{\mathrm{q}}_2{}^2\,)\,cos\;q_2 \\ \dot{\mathrm{q}}_1{}^2\;sin\;q_2 \end{bmatrix} \tag{3.4}$$

In which $\overline{l_2}^2 = {l_1}^2 + {l_1}^2 + 2\,l_1 l_2 \cos\;q_2$

Where q_1 and q_2 are joint positions, for the first and second link. The equation of robot dynamics (3.2) can be written in linear regression form which is often used for the derivation of the adaption law.

$$\tau = \mathrm{Y}(\mathrm{q}, \dot{q}, \ddot{\mathrm{q}})P \tag{3.5}$$

Where, $Y \epsilon R^{nxn}$ is the regressor matrix while $P \epsilon R^{nx1}$ is the vector of parameters.

$$\begin{aligned} &\mathrm{Y}(\mathrm{q}, \dot{q}, \ddot{\mathrm{q}}) \\ &= \begin{bmatrix} \ddot{q}_1 & 2\ddot{q}_1 + \ddot{q}_2 + 2\ddot{q}_1 cos(q_2) - 2\dot{\mathrm{q}}_1\,\dot{\mathrm{q}}_2 sin\;q_2 - \dot{\mathrm{q}}_2{}^2 sin\;q_2 + \ddot{q}_2 cos\;q_2 \\ 0 & \ddot{q}_1 + \ddot{q}_1 cos(q_2) + \ddot{q}_2 + \dot{\mathrm{q}}_1{}^2 sin\;q_2 \end{bmatrix} \end{aligned} \tag{3.6}$$

$$P = \begin{bmatrix} m_1 l^2 \\ m_2 l^2 \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} \tag{3.7}$$

The joint position values for the first and second joint are given in radians as q_1 and q_2, respectively. The concentrated masses are m_1 and m_2. this manipulator, model is developed with the assumption that the links are rigid bodies.

3.2.3 Model of Robot Manipulator-Form II

(3.1) for the rigid robot dynamics can also be written in another form (3.2), which is going to be used throughout this thesis. The robot dynamics given by (3.1) is often used in the literature for the control of robot manipulators. It utilizes the fact that with the proper definition of h, the matrix $\dot{M} - 2h$ is a skew-symmetric matrix [20]. Moreover, matrix h has some additional properties which are going to be presented in this section later. It is important to mention that in form II, h is a matrix while in form I, it is a vector.

$$\tau = M(q)\ddot{q} + h(q, \dot{q}) - J^T(q)\, F_{ext}(t) \tag{3.8}$$

It is important to note that these two forms represent identical systems, and that from II can be derived from form I by usage of Christoffel symbol [139]. The inertial matrix given by (3.3) can be written as:

$$M(q) = \begin{bmatrix} m_1\, {l_1}^2 + m_2\, \overline{l_2}^2 & m_2 l_2\, (l_2 + l_1 \cos\, q_2) \\ m_2 l_2\, (l_2 + l_1 \cos\, q_2) & m_2 {l_2}^2 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \tag{3.9}$$

While the term representing Coriolis and centrifugal forces is now a $h\epsilon R^{2x2}$ matrix where the elements h_{ij} has to be derived.

$$h(q, \dot{q}) = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \tag{3.10}$$

The elements of the above matrix can be obtained by using the following equation and known elements of the inertial matrix M.

$$h_{ij} = \frac{1}{2}\sum_{k=1}^{n} \frac{\partial M_{i,j}}{\partial q_k}\dot{q}_k + \frac{1}{2}\sum_{k=1}^{n}\left(\frac{\partial M_{i,k}}{\partial q_j} - \frac{\partial M_{j,k}}{\partial q_i}\right)\dot{q}_k \tag{3.11}$$

In the case of two DOF robot $n = 2.$ Partial derivatives of the inertial matrix M are,

$$\frac{\partial M_{11}}{\partial q_1} = \frac{\partial M_{12}}{\partial q_1} = \frac{\partial M_{21}}{\partial q_1} = \frac{\partial M_{22}}{\partial q_1} = 0 \tag{3.12}$$

$$\frac{\partial M_{11}}{\partial q_2} = -2P_2 sinq_2 \tag{3.13}$$

$$\frac{\partial M_{12}}{\partial q_1} = \frac{\partial M_{21}}{\partial q_1} = -P_2 sinq_2 \tag{3.14}$$

$$\frac{\partial M_{22}}{\partial q_1} = 0 \tag{3.15}$$

Hence elements of matrix h are:

$$h_{11} = \frac{1}{2}\left(\frac{\partial M_{11}}{\partial q_1}\dot{q}_1 + \frac{\partial M_{11}}{\partial q_2}\dot{q}_2\right) + \frac{1}{2}\left[\left(\frac{\partial M_{11}}{\partial q_1} - \frac{\partial M_{11}}{\partial q_1}\right)\dot{q}_1 + \left(\frac{\partial M_{12}}{\partial q_1} - \frac{\partial M_{12}}{\partial q_2}\right)\dot{q}_2\right] \quad (3.16)$$

$$h_{12} = \frac{1}{2}\left(\frac{\partial M_{12}}{\partial q_1}\dot{q}_1 + \frac{\partial M_{12}}{\partial q_2}\dot{q}_2\right) + \frac{1}{2}\left[\left(\frac{\partial M_{11}}{\partial q_2} - \frac{\partial M_{21}}{\partial q_1}\right)\dot{q}_1 + \left(\frac{\partial M_{12}}{\partial q_2} - \frac{\partial M_{22}}{\partial q_1}\right)\dot{q}_2\right] \quad (3.17)$$

$$h_{21} = \frac{1}{2}\left(\frac{\partial M_{21}}{\partial q_1}\dot{q}_1 + \frac{\partial M_{21}}{\partial q_2}\dot{q}_2\right) + \frac{1}{2}\left[\left(\frac{\partial M_{21}}{\partial q_1} - \frac{\partial M_{11}}{\partial q_2}\right)\dot{q}_1 + \left(\frac{\partial M_{22}}{\partial q_1} - \frac{\partial M_{12}}{\partial q_2}\right)\dot{q}_2\right] \quad (3.18)$$

$$h_{22} = \frac{1}{2}\left(\frac{\partial M_{22}}{\partial q_1}\dot{q}_1 + \frac{\partial M_{22}}{\partial q_2}\dot{q}_2\right) + \frac{1}{2}\left[\left(\frac{\partial M_{21}}{\partial q_2} - \frac{\partial M_{21}}{\partial q_2}\right)\dot{q}_1 + \left(\frac{\partial M_{22}}{\partial q_2} - \frac{\partial M_{22}}{\partial q_2}\right)\dot{q}_2\right] \quad (3.19)$$

Accordingly matrix h in form II is given with the following equation,

$$h\,(\mathrm{q}\,,\dot{\mathrm{q}}\,) = m_2\, l_1\, l_2 \begin{bmatrix} -\,(2\,\dot{\mathrm{q}}_1\,\dot{\mathrm{q}}_2 + \dot{\mathrm{q}}_2{}^2\,)\,cos\,q_2 \\ \dot{\mathrm{q}}_1{}^2\,sin\,q_2 \end{bmatrix} + \begin{bmatrix} (m_1 + m_2)\,g\,l_1\,cos\,q_1 + m_2\,g\,l_2\,cos\,(q_1+q_2) \\ m_2\,g\,l_2\,cos\,(q_1+q_2) \end{bmatrix} \quad (3.20)$$

While inertial matrix M is identical to the matrix M from form I. Nonetheless, robot parameters, and it is given by same equation as in form I.

3.3 Properties of Matrices *M* and *h*

If the robot manipulator dynamic model is given by (3.1), then matrices M and h have specific properties which are exploited by a number of adaptive algorithms as part of the proof of stability and during the derivation of the adaptive law. These properties are used in an adaptive algorithm by Lee and Khalil [95], and therefore, it is considered important to present them here. The Lee and Khalil algorithm is one of the algorithms in nonlinear control of robot manipulators.

With a suitable definition of the matrix h accordingly to the (3.1), matrices M and h are not independent, because h is derives from the matrix $M,$ consequently the following skew-symmetric property holds [36].

$$\left(\dot{M} - 2h\right)^T = -\left(\dot{M} - 2h\right) \tag{3.21}$$

Therefore the following equation holds for all the time.

$$\dot{g}^T \left(\frac{1}{2}\dot{M} - C\right)\dot{g} = 0 \tag{3.22}$$

The matrix h has a linear dynamic structure in $\dot{q},$ hence the following equations are valid [140].

$$h(q, g)k = h(q, k)g \tag{3.23}$$

$$h(q, g+\propto k) = h(q, g)+\propto h(q, k) \tag{3.24}$$

Where $\propto$ a constant while $q, g,$ and k are vectors.

And the Jacobian matrix J is both the forms of robot manipulators is given by:

$$J(q) = \begin{bmatrix} -l_1 \sin\ q_1 - l_2 \sin\ (q_1 + q_2) & -l_2 \sin\ (q_1 + q_2) \\ l_1 \cos\ q_1 + l_2 \cos\ (q_1 + q_2) & l_2 \cos\ (q_1 + q_2) \end{bmatrix} \tag{3.25}$$

3.4 Summary

In this chapter, we have seen the dynamics of the robot manipulators, two general forms of modelling of robot manipulators are presented and basic properties of the inertial matrix *M*, and the *h* matrix which includes Coriolis, centripetal and gravitational forces are mentioned. In our work we will make use of model of manipulator- *form II,* to design the desired control law.

CHAPTER 4

OBSERVERS

4.1 Introduction

The main objective in control systems engineering is to achieve a perfect control signal, which makes the given system to behave in the desired way. This control signal is mainly based on

1. Measured signals.

2. Mathematical model of the plant.

3. Reference signals to be tracked.

Robot manipulators represents a unique class of nonlinear dynamic systems. Many of the controllers used in these dynamic systems considers the location of joints only, and it can be measured using encoder or analyzer-resolver. On the other hand speed measurements can be made using tachometer. Due to the presences of disturbances, sensor errors, parameter uncertainty and modelling errors these measuring devices are obsolete. In the past, plenty of research have been done based on trajectory tracking control of manipulators to enhance the performance and efficiency of the system. Mostly all the existing controllers requires the prior information about the position and velocity measurements for each link involved in the manipulation of robot manipulators. But measurement of velocity requires additional setup which increases the weight as well as the cost of the device also the quality of the measurements is poor which significantly deteriorate the control performance. Therefore many of the robotic manipulators do not provide velocity measurements.

The development of effective robust adaptive controllers signifies the importance of applications of high-performance and high-precision robots. Even in well-structured manufacturing industries, where accuracy and high-speed are bench mark, robots still have to face uncertainty in the parameters, unmodeled dynamic errors. Due to these disturbances it becomes very tough to manipulate accurately the desired position. So, now additionally an observer is designed which helps to tackle all the problems specified above. Luenberger was the first one to start with the design of observer (1964, 1966, and 1971), because of which observers are sometimes referred to as Luenberger observer. According to Luenberger, any system driven by the output of the given system can serve as an observer for that system. Mainly, for all the control methodologies that we have mostly, many of them require full state information which can be sometimes impossible to measure, and sometimes possible but expensive. Also, in some of the control applications, if we are interested in knowing about the state space variable at any instant of time, then we need to estimate the state space variable, and this can be done by dynamical system called the observer connected with the system. The role of the observer is to produce the estimates (good) of the state space variable of the given system. The detectability and observability properties in case of linear systems are closely connected with perfect convergence properties to the existence of the observers but in the case of nonlinear systems the problem of designing an observer has systematic solution and this solution exists only if the nonlinearities are functions of the fully measurable input and the output of the system.

Consider a nonlinear control system

$$\Sigma: \begin{cases} \dot{x} = \mathcal{F}(x,u) & \text{(dynamics of the system)} \\ y = g(x,u) & \text{(output of the system)} \end{cases} \tag{4.1}$$

With state $x \epsilon X$, control $u \epsilon U$ and output $y \epsilon Y$. Here X, U and Y are smooth manifolds of dimension n, p and m respectively. Apart from the measurements the output of the system also includes control inputs. For the given nonlinear system above Σ, an observer can be defined as follows:

Definition 4.1 (Observer): "A dynamical system with state manifold $\mathcal{Z}$, input manifold Y, together with a mapping $\hat{\mathcal{F}}: (\mathcal{Z} \times Y) \rightarrow T\mathcal{Z}$ is an observer for the system Σ, if there exists a smooth mapping $\psi: X \rightarrow \mathcal{Z}$, such that the diagram shown in Figure. 4-1 (the dashed arrow excluded), commutes. The observer gives a full state reconstruction if there in addition is a mapping $\Phi: (\mathcal{Z} \times Y) \rightarrow X$ such that the full diagram in Figure. 4-1 is commutative. (cf. [141] and [142])".

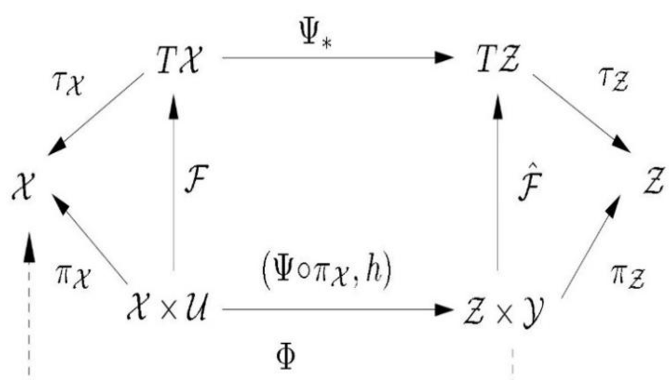


Figure 4-1 Commutative diagram defining an observer [3].

Here, ψ_* denotes the tangent mapping, Π is projection upon a Cartesian factor, while Γ denotes the projection of the tangent bundle. From the definition 4.1, it is clear that the main objective behind designing an observer is to track $\psi(x)$ rather than x. from the definition we can derive the following properties:

Property 4.1. $z(t_0) = \psi\big(x(t_0)\big)$ at some time instance t_0, yields $z(t) = \psi(x(t))$ for all $t \geq t_0$.

Proposition 4.1. For any given observer the property 4.1 is true if and only if the Figure. 4-1 is commutative.

Proof: let us assume the property we have i.e., $z(t) = \psi\big(x(t)\big)$,$\forall t \geq t_0$.

Now let us calculate the time derivative of the above equation

$$\dot{z} = \frac{\partial \psi}{\partial x}\dot{x} = \frac{\partial \psi}{\partial x}\mathcal{F}(x, u) \tag{4.2}$$

Comparing the above equation with the dynamic equation of the system, we have

$$\frac{\partial \psi}{\partial x}\mathcal{F}(x, u) = \hat{\mathcal{F}}(\psi(x), g(x, u)) \tag{4.3}$$

the above equation explains commutative property which was exactly what Figure. 4-1 implies.

Let us now make an assumption that the Figure. 4-1 commutes and $z(t_0) = \psi\big(x(t_0)\big)$ for some $t_0 \epsilon \mathbb{R}^+$. Now solving the differential equation governing z is

$$\begin{gathered}(t) - z(t_0) = \int_{t_0}^{t} \hat{\mathcal{F}}(z, y)d\tau = \int_{t_0}^{t} \frac{\partial \psi}{\partial x}\dot{x}d\tau = \big[\psi\big(x(\tau)\big)\big]_{t_0}^{t} = \\ \psi\big(x(t)\big) - z(t_0)\end{gathered} \tag{4.4}$$

We have the second equality results from the first assumption made and the last inequality from the second assumption made.

Property 4.1 clearly explains the definition 4.1 and also the minimum requirements which should be fulfilled by an observer. Also from this equation it is clear that the definition does not impose restrictions on the convergence of the observers and this serves as the basic work of Luenberger [143]. Convergence rate, domain of attraction, domain of operation etc., must also be specified when these observers are used for practical purposes.

4.2 Properties of Observers

Let us here discuss some of the basic properties of observer that are as follows:

1. Domain of attraction.

2. Rate of convergence.

3. Input dependent convergence.

4.2.1 Domain of Attraction

For an observer, domain of attraction refers to a set of all initial points for which it converges. From this characteristic we can tell that how far from the original condition the initial estimation can be assumed without effecting the convergence properties of the given observer i.e., with the help of this property we able to limit the distance between $\psi(x(t_0))$ and $z(t_0)$ in $\mathcal{Z}$-space. Domain of attraction can be further be divided into global domain of attraction, local domain of attraction and semi-global domain of attraction respectively.

Global domain of attraction: when the observer converges for all $z(t_0)\epsilon\mathcal{Z}$ then it is referred to as global domain of attraction, which can also mean that there is no limit or restriction on z and $\mathcal{Z}$ is the region of contraction [144].

Local domain of attraction: for all $x(t_0)\epsilon\mathcal{X}$, there is a $\varepsilon > 0$ such that the observer is convergent for all $z(t_0)\epsilon\mathbb{B}(\psi(x(t_0)), \varepsilon)$. Here, $\mathbb{B}(\psi(x(t_0)), \varepsilon)$ denotes the ε-ball, centered at $\psi(x(t_0))$.

Semi-global domain of attraction: It refers to class of points within which the observers guarantees convergence. For examples we can design an observer so that convergence is guaranteed for all $z(t_0)\epsilon\mathbb{B}(\psi(x(t_0)), \varepsilon)$.

4.2.2 Rate of Convergence

It refers to the rate at which the estimated error of the system dies down to zero. There are many types of convergence rate such as asymptotic, exponential and finite-time convergence rate. When the observer has asymptotic and exponential rate of decay they are referred to as asymptotic and exponential observers respectively. We must also know that using Lipschitz vector fields it is not possible to find finite-time convergence rate.

4.2.3 Input Dependent Convergence

Generally for a forced nonlinear class of systems, the property of convergence pertaining to the observers are input dependent. In practical applications there are some cases where this strong assumption do not hold true resulting in the non-convergence of the observer for given control inputs i.e., in unicycle robot model case the observer fails to converge.

4.3 Linear Systems Observer Design

Let the given linear system be of the form

$$\begin{cases} \dot{x} = Cx + Du \\ y = Fx \end{cases} \tag{4.5}$$

An observer for the given system can be constructed under the detectability and observability assumptions on $[C, F]$ pair as

$$\begin{cases} \dot{\hat{x}} = C\hat{x} + Du + K(y - \hat{y}) \\ \hat{y} = F\hat{x} \end{cases} \tag{4.6}$$

Here 'K' implies the gain matrix chosen such that $(C - KF)$ is Hurwitz and $K(y - \hat{y})$ is linear output injection, then the dynamics of error are asymptotically stable, that is $\lim_{t\to\infty} \hat{x}(t) = x(t)$.

4.4 Nonlinear Systems Observer Design

Let the given nonlinear system with output injection be

$$\begin{cases} \dot{x} = Cx + f(y, u) \\ y = Dx \end{cases} \tag{4.7}$$

Here f is the nonlinear function which depends on the control signal and measurable output. Design of an observer for the given system can be written as

$$\begin{cases} \dot{\hat{x}} = C\hat{x} + f(y, u) + K(y - \hat{y}) \\ \hat{y} = D\hat{x} \end{cases} \tag{4.8}$$

All the nonlinear systems can't be expressed in form (4.7), we make use of invertible state transformation $\chi = S(x)$ to convert the system back to desired form. The convergence $\lim_{t\to\infty} \hat{\chi} = \chi$ then implies $\lim\limits_{t\to\infty} \hat{x} = x$. In [82], Thau used the below nonlinear system given as

$$\begin{cases} \dot{x} = Cx + f(x,u,t) + \phi(y,u,t) \\ y = Dx \end{cases} \tag{4.9}$$

For the observer

$$\begin{cases} \dot{\hat{x}} = C\hat{x} + f(\hat{x},u,t) + \phi(y,u,t) + L(y-\hat{y}) \\ \hat{y} = D\hat{x} \end{cases} \tag{4.10}$$

Here the nonlinear function $f(x,u,t)$ is Lipchitz with respect to the state x. In [145], Arcak considered monotone sector nonlinearities in the unmeasured states for designing an observer. They have considered the system

$$\begin{cases} \dot{x} = Cx + G\psi(Hx) + \varphi(y,u) \\ y = Dx \end{cases} \tag{4.11}$$

And the observer

$$\begin{cases} \dot{\hat{x}} = C\hat{x} + L(y-\hat{y}) + G\psi(H\hat{x} + K(y-\hat{y})) + \varphi(y,u) \\ \hat{y} = D\hat{x} \end{cases} \tag{4.12}$$

The above observer design disintegrate the error dynamics of the given system into linear system in feedback with the nonlinearities.

4.5 Observer Based Control

One of the possible ways to solve the output feedback control problem is using observer-based control techniques. In case of linear systems we make use of separation principle which allows us to decompose the given problem into sub-problems mainly which can be separated as: design of state observer, and state-feedback controller design. But in the case of nonlinear systems this separation principle cannot be implemented, so the state observer design is case of nonlinear systems is generally coupled with the controller design itself.

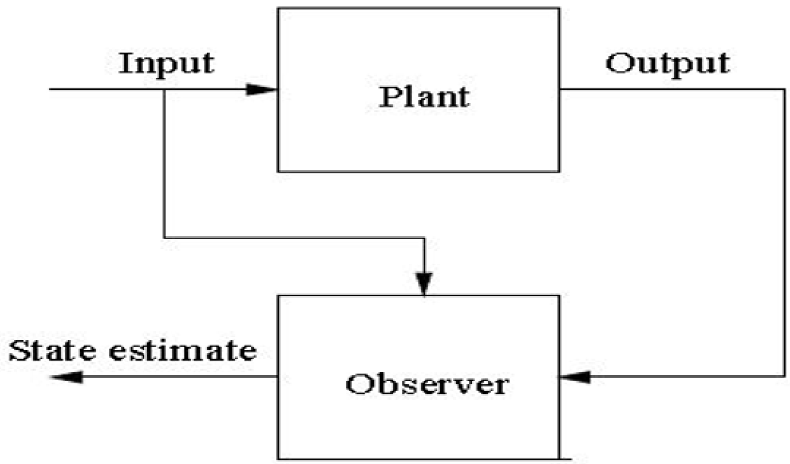


Figure 4-2 Observer scheme [8].

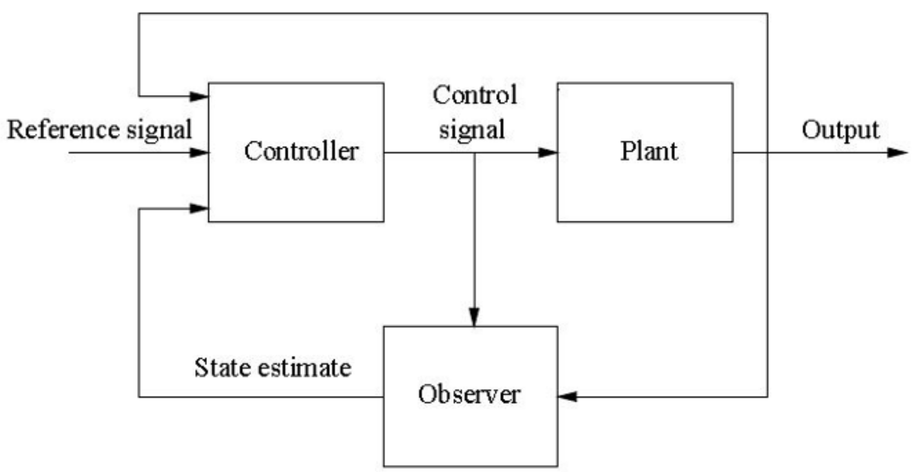


Figure 4-3 Observer-based control scheme [8].

4.5.1 Observer Based State Feedback Design

The whole concept of separation principle is to disintegrate the complete state feedback control design i.e., into full-state feedback part and observer, which is valid for limited class of nonlinear systems and completely valid for all classes of linear systems. The Figure given below shows us the state feedback design using an observer.

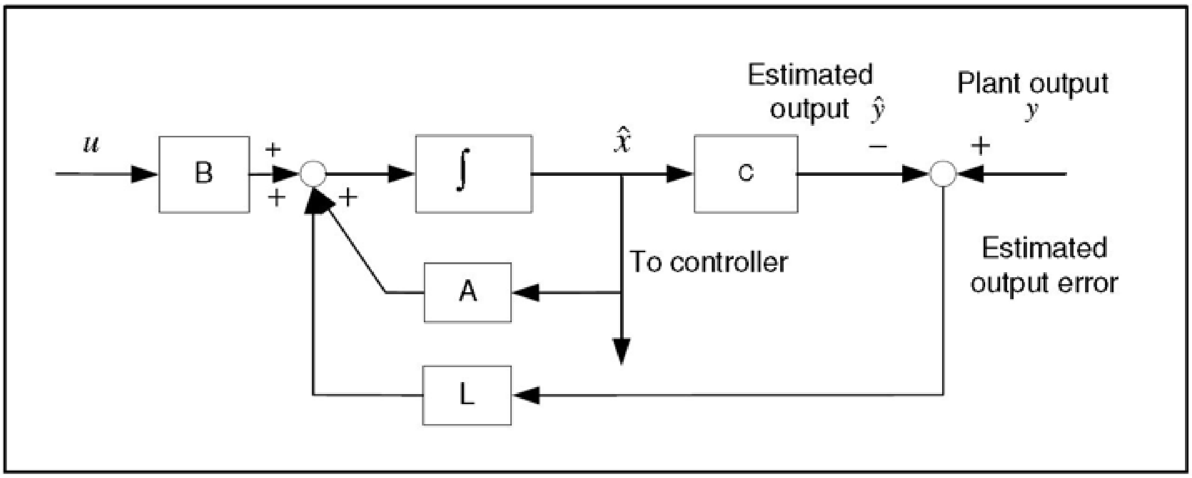


Figure 4-4 State- Feedback Design using an Observer [9].

Let us consider a linear dynamic system which is continuous in time as

$$\begin{cases} \dot{x} = Ax + Gu \\ \quad y = Fx \end{cases} \tag{4.13}$$

The observer design for the given linear system is defined by

$$\dot{\hat{x}} = A\hat{x} + L(y - F\hat{x}) + Gu \tag{4.14}$$

The estimation error is

$$e = x - \hat{x} \tag{4.15}$$

From (4.14) and (4.15) we have

$$\dot{e} = (A - LF)e = \hat{A}e \tag{4.16}$$

If $\hat{A}$ is considered as a stability matrix then we will have convergence of the estimated error to zero. But, when $\hat{A}$ is considered as a constant, then the condition imposed on $\hat{A}$ is its eigenvalues should lie in the left hand side of the given plane. This asymptotic state estimator is known as the Luenberger observer [146]. The matrices A, G and F defined in the above equations cannot be changed, so the only freedom left for designing the observer is the gain matrix L which is carefully determined by the designer.

Some of the possible ways of designing the observer are optimization and pole placement methods. The observer given in the (4.14) resembles the Kalman filter structure, so now we can choose its gain matrix as Kalman filter gain matrix [147], i.e.,

$$L = PF'R^{-1} \tag{4.17}$$

Where P satisfies the Riccati equation given below and it is the estimation error covariance matrix, now let

$$\dot{P} = AP + PA' - PF'R^{-1}FP + Q \tag{4.18}$$

Here R, Q are the positive definite matrix and positive semi-definite matrix respectively. In many of the control applications mostly the steady state covariance matrix is used which is given in (4.17), and this matrix is solved by setting $\dot{P}$ in (4.18) to zero. The final equation resulting from this is termed as algebraic Riccati equation. There are many ways to solve this algebraic Riccati equation and some of the popular algorithms are included in MATLAB and CONTROL-C control system software packages. To make sure that the given gain matrix in (4.14) and (4.15) is optimum, we must necessarily see that the process and observation noise are white with the given matrices Q and R as their spectral densities. In real world practical applications, it is nearly impossible to calculate or determine the spectral density matrix, so these matrices are considered as design parameters which can be manipulated by the designer to a make sure that the overall design objectives are achieved.

We also have an alternate way for finding the observer gain matrix L and solve the given Riccati equation by placing the observer poles [148][149][150], i.e., the eigenvalues of $\hat{A}$ in (4.18). Now from (4.13) the characteristic error equation can be written as

$$\det[sI - (A - LF)] = 0 \tag{4.19}$$

From the above equation L can be chosen such that *A-LF* has stable eigenvalues, with this selection the error *e* will die down to zero and remain there irrespective of the *e(0)* which is the initial condition, and independent of *u(t)* which is the known force function. However we can also choose L such that error remains very small even when the system is subjected to modelling and disturbance input.

Now let us specify the estimator error poles location as

$$s_i = \beta_1, \beta_2, \ldots \beta_n, \tag{4.20}$$

So the characteristic equation for the estimator will be given as

$$\alpha_e(s) = (s - \beta_1)(s - \beta_2) \ldots . (s - \beta_n) \tag{4.21}$$

Comparing (4.20) and (4.21) coefficients we can solve for L as

$$u = -M\hat{x} \tag{4.22}$$

Where

$$\hat{x} = x - e \tag{4.23}$$

Initially, the dynamics of the closed loop is written as

$$\dot{x} = Ax - GM(x - e) \tag{4.24}$$

When we use an observer of full-order then

$$\dot{e} = \hat{A}e = (A - LF)e \tag{4.25}$$

Finally, the complete dynamics of the closed-loop system is given as

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A - GM & GM \\ 0 & A - LF \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} \tag{4.26}$$

Let us suppose

$$\bar{A} = \begin{bmatrix} A - GM & GM \\ 0 & A - LF \end{bmatrix} \tag{4.27}$$

$$|sI - \bar{A}| = |sI - A + GM||sI - A + LF| = 0 \tag{4.28}$$

$A - GM$ are the closed-loop eigenvalues of the full-state nonlinear feedback control system and $A - LF$ are the eigenvalues of the observer.

4.6 High Gain Observers

For designing output feedback controllers, high gain observers are used because of its effectiveness to estimate derivatives (differentiation of the output variables) of the output and the unmeasured states, with special ability of accounting asymptotically attenuating disturbances [151][152][153][154][155][156]. Esfandiari and Khalil were the first one to introduce this technique and from there this technique is used in nearly 60 papers. Let us consider a nonlinear system of second order given as:

$$\dot{x}_1 = x_2 \tag{4.29}$$

$$\dot{x}_2 = \phi(x, u) \tag{4.30}$$

$$y = x_1 \tag{4.31}$$

Supposing that $u = \gamma(x)$ is a state feedback control that stabilizes the origin $x = 0$ of the given second order closed-loop system, the following observer is used

$$\dot{\hat{x}}_1 = \hat{x}_2 + h_1(y - \hat{x}_1) \tag{4.32}$$

$$\dot{\hat{x}}_2 = \phi_0(\hat{x}, u) + h_2(y - \hat{x}_1) \tag{4.33}$$

Where, nominal model $\phi(x, u)$ of the nonlinear function is $\phi_0(\hat{x}, u)$. The equation of estimated error is given as

$$\dot{\tilde{x}}_1 = -h_1\tilde{x}_1 + \tilde{x}_2 \tag{4.34}$$

$$\dot{\tilde{x}}_2 = -h_2\tilde{x}_1 + \delta(x, \tilde{x}) \tag{4.35}$$

where

$$\delta(x, \tilde{x}) = \phi\big(x, \gamma(\tilde{x})\big) - \phi_0(\hat{x}, \gamma(\tilde{x})) \tag{4.36}$$

$$\tilde{x} = \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} = \begin{bmatrix} x_1 - \hat{x}_1 \\ x_2 - \hat{x}_2 \end{bmatrix} \tag{4.37}$$

The observer gain for any asymptotic observer given as

$$H = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \tag{4.38}$$

is designed such that the asymptotic error convergence is achieved, i.e., $\lim_{t \to \infty} \tilde{x}(t) = 0$. For example when the disturbance term $\delta(x, \tilde{x})$ is absent, then the convergence of the asymptotic estimated error is evaluated by building A_0 which is the observer gain matrix such that

$$A_0 = \begin{bmatrix} -h_1 & 1 \\ -h_2 & 0 \end{bmatrix} \tag{4.39}$$

is Hurwitz. Form the above equation we can say that for any positive value of h_1, h_2 the second order A_0 system is Hurwitz.

Now if the disturbance $\delta(x, \tilde{x})$ is present inside the system, then the observer gain is designed in such a way so as to cancel the effect of the disturbance δ on the asymptotic estimation error $\tilde{x}$. This can be made possible only when transfer function from δ to $\tilde{x}$ be identically zero. So now the observer gain can be designed as $h_1 \gg h_2 \gg 1$, such that the closed loop transfer function $H_0(s)$ is ideally converging to zero.

$$H_0(s) = \frac{1}{s^2+h_1 s+h_2}\begin{bmatrix} 1 \\ s+h_1 \end{bmatrix} \tag{4.40}$$

Where

$$h_1 = \frac{\alpha_1}{\varepsilon}, h_2 = \frac{\alpha_2}{\varepsilon^2} \tag{4.41}$$

For some positive constants $\alpha_1, \alpha_2$and ε with $\varepsilon \ll 1$, $\lim_{\varepsilon\to 0} H_0(s) = 0$.

Now let us calculate the scaled error as

$$\eta_1 = \frac{\tilde{x}_1}{\varepsilon}, \eta_2 = \tilde{x}_2 \tag{4.42}$$

So that these variable satisfies the below equation

$$\varepsilon\dot{\eta}_2 = -\alpha_2\eta_1 + \varepsilon\delta(x, \tilde{x}) \tag{4.43}$$

$$\varepsilon\dot{\eta}_2 = -\alpha_2\eta_1 + \varepsilon\delta(x, \tilde{x}) \tag{4.44}$$

From the above equations it is clear that in order to reduce the effect of disturbance $\delta(x, \tilde{x})$ we need to reduce ε, and also the dynamics of x, will be lower than the dynamics of the error. But from (4.44) the change of variable done for designing observer may cause $\eta_1(0)$ which iss the initial condition to be of order $O\left(\frac{1}{\varepsilon}\right)$, even when $\tilde{x}_1(0)$ is of order O(1). Now from the above initial condition the solution of (4.44) will have term of exponential form as $\left(\frac{1}{\varepsilon}\right)e^{\frac{-at}{\varepsilon}}$ for some a> 0, this exponential term exhibits an impulsive-like behavior and transients peaks to $O\left(\frac{1}{\varepsilon}\right)$ before reaching or decaying to zero. This phenomenon is known as peaking behavior, and this is an intrinsic feature of any high gain observer design which rejects disturbance $\delta(x, \tilde{x})$ effect from (4.42); i.e., for any design with $h_1 \gg h_2 \gg 1$. When the impulse like behavior is transferred from high gain observer to the system resulting in peak phenomenon which destabilizes the given system.

The high gain observer can also be viewed as an approximate differentiator, and this can be seen in some cases where the nominal function ϕ_0 is selected to be zero; and for this case the observer designed is linear. For the nonlinear observer given (4.40) the transfer function from y to $\hat{x}$ can be calculated by

$$\eta_1 = \frac{\tilde{x}_1}{\varepsilon}, \eta_2 = \tilde{x}_2 \tag{4.45}$$

Realizing that high gain observer can work as an approximate differentiator, and also practically limiting how small ε could be when the unmodeled high frequency sensor and measurement noise are included inside the system. The combination of high gain observers with state feedback control allows for a separation phenomenon and in order to meet the required objectives the feedback control is designed first. The high gain observer is able to achieve and quickly recover from the performance which is achieved when the system was under state feedback. Almost all the papers which are based on high gain observer employs this separation approach.

The high gain observers has been recently applied in many nonlinear systems. Henrik Rehbinder and Xiaoming Hu [157] uses high gain observers to estimate the nonlinear angles of a moving robot i.e., roll and pitch angles respectively. Xiaoming Oh and Hassan K.Khalil [154] used high gain observers along with variable structure control to discover the use of nonlinear feedback tracking. E.S.Shin and K.W.Lee [117] were the first one to design a robust output feedback control of robot manipulators. J.De Leon, K Busawon, and G.Acosta [158] were the first one to use the digital implement the high gain observer based control of rigid robot. Nael H. El-Farra and Pamagiotis D.Christofides [159] designed high gain observers for a large class of nonlinear process involving uncertain variables and also actuator constraints with a bounded robust and optimal state feedback controller. This high gain observers are used along with robust adaptive control scheme in order to control robot manipulator.

CHAPTER 5

OBSERVER BASED ROBUST ADAPTIVE CONTROL OF UNCERTAIN ROBOT MANIPULATOR

5.1 Introduction

In this section of the thesis we proposes an output feedback robust adaptive control and observer design of nonlinear systems, and in particular of robot manipulator. It mainly focuses on the high performance robust adaptive control of robot manipulator in the presence of parametric uncertainties and uncertain nonlinearities (disturbances). It order to achieve the above target for the minimization of model uncertainties, unmodeled dynamics and external force disturbance effect to control the robot manipulator an observer based robust adaptive tracking control scheme is then developed. In the control design, no considerations are required for the upper bound of system uncertainties and disturbances. Also, the speed of variation and the magnitude of unknown parameters and perturbations are assumed to have no limitations. The controller in this work uses an adaptation mechanism for both high gain time-varying and constant non-linear observer along with simplicity and universality properties to ensure robust tracking and make the system follow desired reference model.

Unlike the previous approaches as discussed before, this method is rich in the sense that:

1. It does not require the skew symmetric property for the controller design,

2. The unknown system parameters are not required to be slow or bounded,

3. Uncertainties in inertia are taken into account in an unstructured form,

4. The external force vector is also considered as time varying vector with unknown bound.

5.2 Problem Formulation

Consider the dynamic equations of robot manipulator as

$$M(q)\ddot{q} + h(q, \dot{q}) = \tau + J^T(q)\, F_{ext}(t) \tag{5.1}$$

Table 2 Parameters of robot manipulator

$q \in R^n$	vector of angular joint positions
$\dot{q} \in R^n$	vector of angular velocity
$\ddot{q} \in R^n$	vector of angular acceleration
$M(q) \in R^{nxn}$	positive definite inertia matrix
$h(q, \dot{q}) \in R^n$	includes the Coriolis, centripetal and gravitational forces
$\tau \in R^n$	applied torque to be designed
$J(q)$	Jacobian matrix with appropriate dimension
$F_{ext}(t)$	external force (and moment) vector which is exerted on the end effector

In the above equation, $M(q)$, $h(q, \dot{q})$, $F_{ext}(t)$ are unknown and this values can be calculated as:

Property 5.1: The inertia matrix is decomposed as

$$M\ (q) = M_0(q) + M_\Delta(q), \tag{5.2}$$

In which $M_0(q)$ denotes the known nominal part, and $M_\Delta(q)$ is a norm-bounded unmodelled perturbation with bounded time derivative, i.e., $\|M_\Delta(q)\| \le \alpha_1$ and $\|\dot{M}_\Delta(q)\| \le \alpha_2$ where α_1, α_2 are unknown constants.

Property 5.2: Due to the variations in such parameters included in matrix $h(q\ ,\dot{q}\ )$ it can be expressed in a general form as

$$h(q\ ,\dot{q}\ ) = h_0(q\ ,\dot{q}\ ) + \Phi(q\ ,\dot{q}\ )\ \Theta + h_\Delta(q\ ,\dot{q}\ ) \tag{5.3}$$

Where, $h_0(q\ ,\dot{q}\ )$ is a known vector, Θ represents the vector of uncertain parameters, $\Phi(q\ ,\dot{q}\ )$ is a dimensionally compatible matrix, and $h_\Delta(q\ ,\dot{q}\ )$ is the vector of unstructured uncertainties such that $\|h_\Delta(q\ ,\dot{q})\| \le \alpha_3$, where α_3 is an uncertain parameter.

Property 5.3: In general, the external force disturbance $F_{ext}(t)$ is time-varying, with unknown bound, i.e., $\|F_{ext}(t)\| \le \eta$ where η is unknown.

Remark 5.1: From the properties 5.1-5.3, we almost include all kinds of disturbances and uncertainties occurred due to numerous applications of robot manipulators in different conditions. The basic objective here is to track the twice differentiable desired trajectory $q_r(t)$ in the presence of uncertainties, unknown time-varying parameters and external force disturbances.

Remark 5.2: Unlike the most previous works [107], [160]–[163] the skew symmetric property is not required here.

5.3 Robust Adaptive Controller Design

In this section, robust adaptive tracking controllers are developed for robot manipulators (5.1) ensuring robustness with respect to various kinds of model uncertainties and external disturbances.

As a preliminary step to develop the controller, define the tracking error as,

$e = q_r(t) - q(t)$, and two error metric functions

$$S(t) = \dot{e}(t) + \Lambda e(t) \tag{5.4}$$

$$S_r(t) = \ddot{q}(t) + \Lambda \dot{e}(t) \tag{5.5}$$

Where Λ is positive definite matrix, based on the notations used in properties 5.1 and 5.2, define positive parameter α=max$\{\alpha_1, \alpha_2, \alpha_3\}$. The applied torque input is proposed here as

$$\tau = h_0 + M_0\, S_r + \left[0.5\, \dot{M}_0 + K\right] S + \tau_{a1} + \tau_{a2} \tag{5.6}$$

Where K is a positive definite matrix. Moreover,τ_{a1} deals with unstructured uncertainties, and τ_{a2}, tackles the unknown parameters and disturbances, are adaptive sub controllers to be designed.

Theorem 5.1: For uncertain robotic systems, described by (5.1) with properties 5.1-5.3, the smooth bounded reference trajectory $q_r(t)$ is given. Consider the control law (5.6) with

$$\tau_{a1} = \hat{\alpha}\, S \left[\frac{1}{2} + \hat{\alpha} \frac{1}{\|S\|\hat{\alpha} + \frac{1}{3}\delta\, e^{-\sigma t}} + \hat{\alpha} \frac{{S_r}^T S_r}{\|S_r\|\, \|S\|\, \hat{\alpha} + \frac{1}{3}\delta\, e^{-\sigma t}}\right] \tag{5.7}$$

$$\tau_{a2} = \Phi(q, \dot{q})\,\widehat{\Theta} + \hat{\eta}^2 \frac{J^T J\, S}{\|S^T J^T\|\hat{\eta} + \frac{1}{3}\delta\, e^{-\sigma t}} \tag{5.8}$$

Where δ, and σ are two (small) positive constants specified by the designer to avoid discontinuity. Moreover, $\widehat{\alpha}$, $\hat{\eta}$ and $\widehat{\Theta}$ are updated with adaptation laws

$$\dot{\hat{\alpha}}_{dot} = \gamma_\alpha \left[(\|S_r\| + 1)\|S\| + \frac{1}{2} S^T S \right] \tag{5.9}$$

$$\dot{\hat{\eta}}_{dot} = \gamma_\eta \|S^T J^T\| \tag{5.10}$$

$$\dot{\widehat{\Theta}}_{dot} = \Gamma\, \Phi^T(q, \dot{q})\, S \tag{5.11}$$

In which γ_α and γ_η are the adaptation gains and $\Gamma = \Gamma^T > 0$ is the adaptation matrix. The robust adaptive controller formed by (5.6)-(5.11), ensures the convergence of tracking error, despite the perturbations.

5.4 Stability Analysis of the Proposed Controller

Choose the Lyapunov function

$$U\left(e, \dot{e}, \widetilde{\alpha}, \widetilde{\eta}, \widetilde{\Theta}\right) = V(e, \dot{e}) + \frac{1}{2\gamma_\alpha}\tilde{\alpha}^2 + \frac{1}{2\gamma_\eta}\tilde{\eta}^2 + \frac{1}{2}\widetilde{\Theta}^T \Gamma^{-1} \widetilde{\Theta} \tag{5.12}$$

With

$$V(e, \dot{e}) = e^T K e + \frac{1}{2} S^T M S \tag{5.13}$$

Where $\tilde{\alpha} = \alpha - \hat{\alpha}$, $\tilde{\eta} = \eta - \hat{\eta}$, and $\widetilde{\Theta} = \Theta - \widehat{\Theta}$ denote the parameter estimation errors. The time derivative of (5.13) is

$$\dot{V} = 2e^T K \dot{e} + S^T(M\ddot{e} + M\dot{e}) + \frac{1}{2} S^T \dot{M} S \tag{5.14}$$

Substituting $\ddot{e}$ by $\ddot{q}_r - \ddot{q}$ in (c) and then replacing $M\ddot{q}$ from (5.1), one can obtain

$$\dot{V} = 2e^T K\dot{e} + S^T\left(M\ddot{q}_r + h - \tau - J^T F_{ext} + M\dot{e} + \frac{1}{2}\dot{M}S\right) \tag{5.15}$$

Replacing the uncertain matrix $M(q)$ and unknown vector $h(q,\dot{q})$ from (5.2) and (5.3) respectively, and incorporating the control law (5.6) into (5.15) yields

$$\begin{aligned}\dot{V} &= 2e^T K\dot{e} - S^T KS \\ &\quad + S^T\Big(M_\Delta(\ddot{q}_r + \dot{e}) + \frac{1}{2}\dot{M}_\Delta S + \Phi(q\,,\dot{q}\,)\,\Theta + h_\Delta(q,\dot{q}) \\ &\quad - J^T F_{ext}(t) - \tau_{a1-}\tau_{a2}\Big)\end{aligned} \tag{5.16}$$

Now, taking into account properties 5.1-5.3 implies that

$$\begin{aligned}\dot{V} \le -e^T Ke - \dot{e}^T K\dot{e} + \alpha_1\|S_r\|\|S\| + \tfrac{1}{2}\alpha_1 S^T S + \alpha_3\|S\| + \\ S^T\Phi(q\,,\dot{q}\,)\,\Theta - S^T J^T F_{ext}(t) - S^T\tau_{a1} - S^T\tau_{a2}\end{aligned} \tag{5.17}$$

$$\begin{aligned}\dot{V} \le -e^T Ke - \dot{e}^T K\dot{e} + \alpha\left(\frac{1}{2}S^T S + \|S_r\|\|S\| + \|S\|\right) + S^T\Phi(q\,,\dot{q}\,)\,\Theta \\ + \eta\|S^T J^T\| - S^T\tau_{a1} - S^T\tau_{a2}\end{aligned} \tag{5.18}$$

By adaptive sub-controllers (5.10) and (5.11), one obtains

$$-S^T\tau_{a1} \le -\tfrac{1}{2}\hat{\alpha}S^T S - \hat{\alpha}\|S\| - \hat{\alpha}\|S_r\|\|S\| + \tfrac{2}{3}\delta e^{-\sigma t} \tag{5.19}$$

$$-S^T\tau_{a2} \le -S^T\Phi(q\,,\dot{q}\,)\,\widehat{\Theta} - \hat{\eta}\|S^T J^T\| + \frac{1}{3}\delta e^{-\sigma t} \tag{5.20}$$

Incorporating the inequalities (5.19) and (5.20) into (5.18) gives

$$\begin{aligned}\dot{V} \le -e^T Ke - \dot{e}^T K\dot{e} + \tilde{\alpha}\left(\frac{1}{2}S^T S + \|S_r\|\|S\| + \|S\|\right) + \tilde{\eta}\|S^T J^T\| \\ + S^T\Phi(q\,,\dot{q}\,)\,\widetilde{\Theta}\end{aligned} \tag{5.21}$$

On the other hand, the time derivative of (5.12) is calculated as

$$\dot{U} \le \dot{V} - \frac{1}{\gamma_\alpha}\tilde{\alpha}\dot{\hat{\alpha}} - \frac{1}{\gamma_\eta}\tilde{\eta}\dot{\hat{\eta}} + \frac{1}{2}\dot{\widehat{\Theta}}^T\Gamma^{-1}\widetilde{\Theta} \tag{5.22}$$

Using (5.21) and update laws (5.9)-(5.11) in (5.22) yields

$$\dot{U} \leq -e^T K e - \dot{e}^T K \dot{e} + \delta e^{-\sigma t} \tag{5.23}$$

In order to use barbalat's lemma in the proof procedure, it is shown here that $e(t)$ and $\dot{e}(t)$ are bounded and $e(t)$ is square-integrable.

By (5.23) we can conclude

$$\dot{U} \leq -e^T K e + \delta e^{-\sigma t} \tag{5.24}$$

Which means barbalat's lemma [82] guarantees convergence and also boundedness of closed-loop signals despite the system uncertainties and external force disturbance. The exponential terms, incorporated in adaptive sub controllers (5.7) and (5.8), are to avoid chattering and discontinuity of control input, without violating the convergence property of tracking error and closed loop stability.

5.5 Observer Design

In this section of the paper we develop an output feedback tracking control scheme with time invariant and time-varying observer gains for a two link manipulator whose dynamics are given in (5.1). The observer design is based only on measurement of position, using $x_1 = q \in R^n$ and $x_2 = \dot{q} \in R^n$, now the dynamics of the robot manipulator can be rewritten with change of variable in state space form as:

$$\dot{x}_1 = x_2 \tag{5.25}$$

$$\dot{x}_2 = M^{-1}(x_1)[\tau + J^T(x_1)\, F_{ext}(t) - h(x_1, x_2)] \tag{5.26}$$

$$y = x_1 \tag{5.27}$$

Based on (5.25)-(5.27), the nonlinear observer is designed as:

$$\dot{\hat{x}}_1 = \hat{x}_2 + L_1(x_1 - \hat{x}_1) + L_2(x_2 - \hat{x}_2) \tag{5.28}$$

$$\dot{\hat{x}}_2 = M^{-1}{}_0(\hat{x}_1)\,[-h(\hat{x}_1\,,\hat{x}_2\,) + \tau] + L_3(x_1 - \hat{x}_1) + L_4(x_2 - \hat{x}_2) \qquad (5.29)$$

Where, L_1, L_2, L_3, L_4 are the correction terms & $\hat{x}_1, \hat{x}_2$ are the estimates of x_1, x_2 and the overall diagram for the proposed scheme is shown in Figure. 5-1.

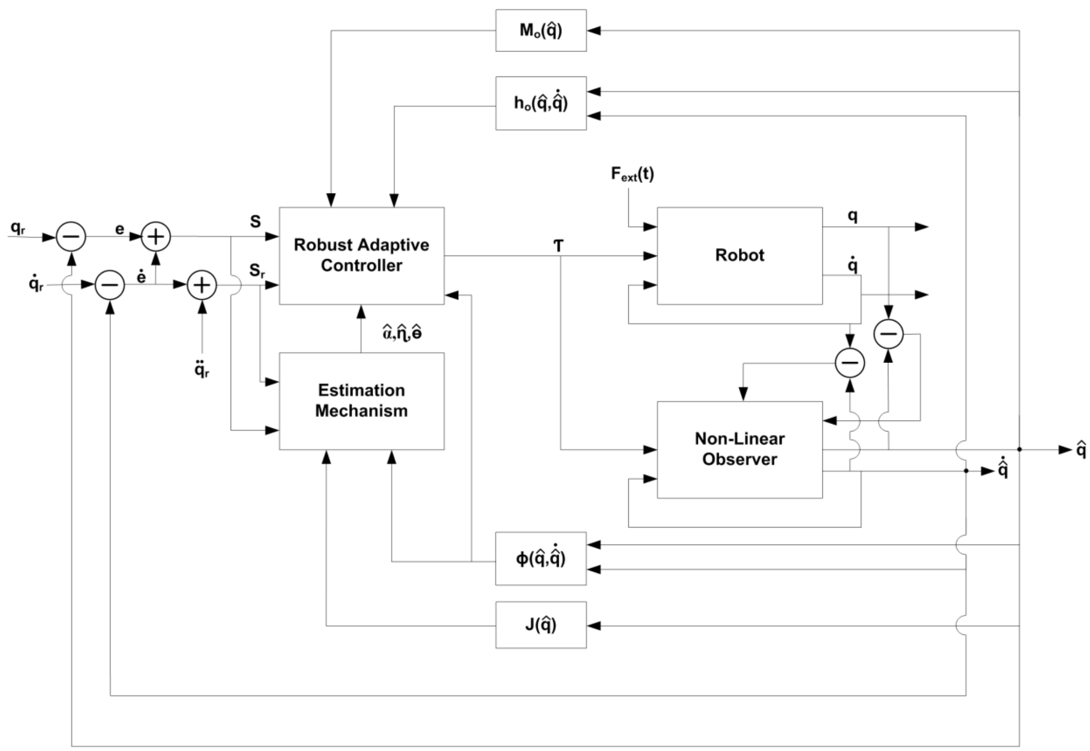


Figure 5-1 The overall diagram of the proposed scheme.

As an initial step in design define the tracking errors as

$$\dot{e}_1 = e_2 - L_1 e_1 - L_2 e_2 \qquad (5.30)$$

$$\dot{e}_2 = -L_3 e_1 - L_4 e_2 - \zeta\varphi(e_1, e_2) + w(t) \qquad (5.31)$$

Where,$e_1 = x_1 - \hat{x}_1$, $e_2 = x_2 - \hat{x}_2$ are error estimates, $w(t)$ is the external force disturbance, ζ is assumed to be unknown vector and $\varphi(e_1, e_2)$ as known nonlinear function. The following theorem states the stability of the nonlinear high gain observer.

Theorem 5.2 for time invariant observer gains: For the uncertain robotic systems described by (5.25)-(5.27), the state observer estimation error as described by (5.28)-

(5.29) is stable and ultimately bounded provided that $L_1 \gg \frac{\lambda}{2}, L_2 > 1, L_3 > 1, L_4 \gg \lambda_1^4$. Where λ, λ_1 are the design parameter selected by the designer. Moreover, the estimate of the uncertain parameter is updated by the following adaptive algorithm.

$$\dot{\hat{\zeta}} = \left[\frac{1}{2\lambda_1^4}\varphi(e_1, e_2)(e_2^T + \lambda_1 e_1^T)\right] \tag{5.32}$$

with $\zeta(t_0)$ is assumed to be close to zero. All the generated state estimates $(\dot{\hat{x}}_1, \dot{\hat{x}}_2)$ are uniformly bounded. Furthermore, when λ_1 is chosen as very high, then the disturbance $w(t)$ can be bounded by $\varepsilon(e_1, e_2)$, such that $\left|\frac{1}{2\lambda_1^4} w(t)[e_2^T + \lambda_1 e_1^T]\right| \leq \varepsilon(e_1, e_2)$. Where $\varepsilon(e_1, e_2) \to 0$ as $t \to \infty$.

Proof: Let us consider Lyapunov candidate function as

$$V(x) = \frac{1}{2}e_1^T e_1 + \frac{1}{2\lambda_1^4}[e_2 + \lambda_1 e_1]^T[e_2 + \lambda_1 e_1] + \frac{1}{2}\zeta_1^T\zeta_1 \tag{5.33}$$

Let $V_1(x) = \frac{1}{2}e_1^T e_1$; The time derivative of the above equation is $\dot{V}_1(x) = e_1^T \dot{e}_1$. Substituting the value of $\dot{e}_1$ in the above equation, we get

$$\dot{V}_1(x) = e_1^T(e_2 - L_1 e_1 - L_2 e_2) \tag{5.34}$$

For the term $e_1^T e_2$ we use young's inequality with $\lambda > 0$. We have $e_1^T e_2 \leq \frac{\lambda e_1^T e_1}{2} + \frac{e_2^T e_2}{2\lambda}$ leading to

$$\dot{V}_1(x) \leq -e_1^T\left[L_1 - \frac{\lambda}{2}\right]e_1 - e_1^T L_2 e_2 + \frac{e_2^T e_2}{2\lambda} \tag{5.35}$$

Now let $V_2(x) = \frac{1}{2\lambda_1^4}[e_2 + \lambda_1 e_1]^T[e_2 + \lambda_1 e_1]$.

The time derivative of the above equation is,

$$\dot{V}_2(x) = \frac{1}{2\lambda_1^4}[e_2^T + \lambda_1 e_1^T]\dot{e}_2 + \frac{1}{2\lambda_1^3}[e_2^T + \lambda_1 e_1^T]\dot{e}_1 \tag{5.36}$$

On replacing $\dot{e}_1$ and $\dot{e}_2$ in the above equation, we have

$$\dot{V}_2(x) = \frac{1}{2\lambda_1^4}[e_2^T + \lambda_1 e_1^T][-L_3 e_1 - L_4 e_2 - \alpha\varphi(e_1, e_2) + w(t)]$$
$$+ \frac{1}{2\lambda_1^3}[e_2^T + \lambda_1 e_1^T][e_2 - L_1 e_1 - L_2 e_2]$$

$$\begin{aligned}\dot{V}_2(x) = &-\frac{1}{2\lambda_1^4}e_2^T\left[L_3 + \lambda_1 L_4 + \lambda_1 L_1 - {\lambda_1}^2 + {\lambda_1}^2 L_2\right]e_1 \\ &- \frac{1}{2\lambda_1^4}e_2^T[L_4 - \lambda_1 + \lambda_1 L_2]e_2 - \frac{1}{2\lambda_1^3}e_1^T[L_3 + \lambda_1 L_1]e_1 \\ &- \frac{1}{2\lambda_1^4}\alpha\varphi(e_1, e_2)[e_2^T + \lambda_1 e_1^T] + \frac{1}{2\lambda_1^4}w(t)[e_2^T + \lambda_1 e_1^T]\end{aligned} \tag{5.37}$$

Finally, let us select $V_3(x) = \frac{1}{2}\zeta_1^T\zeta_1$; and find the time derivative, and that is given as $\dot{V}_3(x) = \dot{\zeta}^T\zeta$.

Combining all the terms for $\dot{V}(x) = \dot{V}_1(x) + \dot{V}_2(x) + \dot{V}_3(x)$. Substitute the values of $\dot{V}_1(x), \dot{V}_2(x), \dot{V}_3(x)$ in the above equation yields

$$\begin{aligned}\dot{V}(x) \le &-\tfrac{1}{2\lambda_1^4}e_2^T\left[L_3 + \lambda_1 L_4 + \lambda_1 L_1 - {\lambda_1}^2 + {\lambda_1}^2 L_2 + 2{\lambda_1}^4 L_2\right]e_1 - \\ &\tfrac{1}{2\lambda_1^4}e_2^T\left[L_4 - \lambda_1 + \lambda_1 L_2 - \tfrac{{\lambda_1}^4}{\lambda}\right]e_2 - \tfrac{1}{2\lambda_1^3}e_1^T\left[L_3 + \lambda_1 L_1 + 2{\lambda_1}^4\left(L_1 - \right.\right. \\ &\left.\left.\tfrac{\lambda}{2}\right)\right]e_1 - \zeta[\tfrac{1}{2\lambda_1^4}\varphi(e_1, e_2)(e_2^T + \lambda_1 e_1^T) - \dot{\zeta}^T] + \tfrac{1}{2\lambda_1^4}w(t)[e_2^T + \lambda_1 e_1^T]\end{aligned} \tag{5.38}$$

The right hand-side of the equation will always be negative with proper selection of the correction terms L_1, L_2, L_3, L_4 and λ,and λ_1.

Remark 5.4: The design parameters λ, λ_1 should be selected as big as possible to make the effect of external disturbance as small as possible, thus stabilizing the system.

Remark 5.5: It is convenient to choose $L_1 \gg \frac{\lambda}{2}, L_2 > 1, L_3 > 1, L_4 \gg \lambda_1^4$ so as to cancel the negative effect of design parameter terms.

Taking the above remarks (5.4)-(5.5) into account, the Lyapunov function (5.38) guarantees the perfect tracking error convergence and also boundedness of closed-loop signals despite uncertainties and external disturbances.

Theorem 5.3 for time varying observer gains: For the uncertain robotic systems described by (5.1), the state observer estimation error as described by (5.30)-(5.31) is stable and ultimately bounded provided that $\mathrm{L}_1 \gg \frac{\lambda}{2}, \mathrm{L}_2 > 1, \mathrm{L}_3 > 1, \mathrm{L}_4 \gg \lambda_1^4(t)$. Where $\lambda, \lambda_1(t)$ are the design parameter selected by the designer. Moreover, the estimate of the uncertain parameter is updated by the following adaptive algorithm.

$$\dot{\zeta}^T = \left[\frac{e_2^T}{\lambda_1^4(t)} + \frac{e_1^T}{\lambda_1^3(t)}\right] \varphi(e_1, e_2)$$

With $\zeta(t_0)$ is assumed to be close to zero. All the generated state estimates $(\dot{\hat{x}}_1, \dot{\hat{x}}_2)$ are uniformly bounded. Furthermore, when $\lambda_1(t)$ is chosen as very high, then the disturbance $w(t)$ can be bounded by $\varepsilon(e_1, e_2)$, such that

$\left|\left[\frac{e_2^T}{\lambda_1^4(t)}+\frac{e_1^T}{\lambda_1^3(t)}\right]w(t)\right| \le \varepsilon(e_1,e_2)$. Where $\varepsilon(e_1, e_2) \to 0$ as $t \to \infty$.

Proof: Let us consider Lyapunov candidate function as

$$V(x)=\frac{1}{2}e_1^Te_1+\frac{1}{2\lambda_1^4(t)}[e_2+\lambda_1(t)e_1]^T[e_2+\lambda_1(t)e_1]+\frac{1}{2}\zeta_1^T\zeta_1 \tag{5.39}$$

Here $\lambda_1$is treated as time-varying design parameter selected by the designer as

$$\lambda_1(t)=\lambda_{max}(1-e^{-\varsigma t})$$

$$\dot{\lambda}_1(t)=-\lambda_{max}(\varsigma(1-e^{-\varsigma t})-\varsigma)$$

$$\dot{\lambda}_1(t)=-\varsigma\lambda_1(t)+\varsigma\lambda_{max} \tag{5.40}$$

Where λ_{max} represents the maximum value of $\lambda_1(t)$and ς is the delay inside the system.

Let $V_1(x)=\frac{1}{2}e_1^Te_1$

The time derivative of the above equation is $\dot{V}_1(x)=e_1^T\dot{e}_1$; substitute the value of $\dot{e}_1$ in the above equation, we get

$$\dot{V}_1(x)=e_1^T(e_2-L_1e_1-L_2e_2) \tag{5.41}$$

For the term $e_1^Te_2$ we use young's inequality with $\lambda>0$. We have $e_1^Te_2\le\frac{\lambda e_1^Te_1}{2}+\frac{e_2^Te_2}{2\lambda}$

Substitute the above in equation (5.39) we get,

$$\dot{V}_1(x)\le-e_1^T\left[L_1-\frac{\lambda}{2}\right]e_1-e_1^TL_2e_2+\frac{e_2^Te_2}{2\lambda} \tag{5.42}$$

Now let $V_2(x)=\frac{1}{2\lambda_1^4(t)}[e_2+\lambda_1(t)e_1]^T[e_2+\lambda_1(t)e_1]$

$$V_2(x)=\frac{1}{2\lambda_1^4(t)}[e_2^Te_2]+\frac{1}{2\lambda_1^4(t)}[2\lambda_1(t)e_1^Te_2]+\frac{1}{2\lambda_1^4(t)}[\lambda_1^2(t)e_1^Te_1] \tag{5.43}$$

And the time derivative of the above equation, can be calculated as,

$$\dot{V}_2(x) = \frac{1}{2}\left[\frac{\lambda_1^4(t)(2e_2^T\dot{e}_2) - e_2^T e_2\big(4\lambda_1^3(t)\dot{\lambda}_1(t)\big)}{(\lambda_1^4(t))^2}\right] + \frac{\lambda_1^3(t)(e_1^T\dot{e}_2 + \dot{e}_1 e_2^T) - e_1^T e_2\big(3\lambda_1^2(t)\dot{\lambda}_1(t)\big)}{(\lambda_1^3(t))^2} + \frac{1}{2}\left[\frac{\lambda_1^2(t)(2e_1^T\dot{e}_1) - e_1^T e_1\big(2\lambda_1(t)\dot{\lambda}_1(t)\big)}{(\lambda_1^2(t))^2}\right] \tag{5.44}$$

Taking like terms together

$$\dot{V}_2(x) = \left[\frac{e_2^T}{\lambda_1^4(t)} + \frac{e_1^T}{\lambda_1^3(t)}\right]\dot{e}_2 + \left[\frac{e_2^T}{\lambda_1^3(t)} + \frac{e_1^T}{\lambda_1^2(t)}\right]\dot{e}_1 - \left[\frac{2e_2^T e_2}{\lambda_1^5(t)} + \frac{3e_1^T e_2}{\lambda_1^4(t)} + \frac{e_1^T e_1}{\lambda_1^3(t)}\right]\dot{\lambda}_1(t) \tag{5.45}$$

On replacing $\dot{e}_1$, $\dot{e}_2$, $\dot{\lambda}_1(t)$ in the above equation we have

$$\dot{V}_2(x) = \left[\frac{e_2^T}{\lambda_1^4(t)} + \frac{e_1^T}{\lambda_1^3(t)}\right](-L_3 e_1 - L_4 e_2 - \zeta\varphi(e_1, e_2) + w(t)\ ) + \left[\frac{e_2^T}{\lambda_1^3(t)} + \frac{e_1^T}{\lambda_1^2(t)}\right](e_2 - L_1 e_1 - L_2 e_2) - \left[\frac{2e_2^T e_2}{\lambda_1^5(t)} + \frac{3e_1^T e_2}{\lambda_1^4(t)} + \frac{e_1^T e_1}{\lambda_1^3(t)}\right]\ (-\varsigma\lambda_1(t) + \varsigma\lambda_{max}) \tag{5.46}$$

$$\dot{V}_2(x) = -\frac{1}{\lambda_1^4(t)} e_2^T\big(L_3 + \lambda_1(t)L_4 + \lambda_1(t)L_1 + \lambda_1^2(t)(L_2 - 1) + 3\varsigma(\lambda_{max} - \lambda_1(t))\big)e_1 - \frac{1}{\lambda_1^4(t)} e_2^T\left[L_4 + \lambda_1(t)(L_2 - 1) + 2\varsigma\left(\frac{\lambda_{max}}{\lambda_1(t)} - 1\right)\right]e_2 - \frac{1}{\lambda_1^3(t)} e_1^T\big(L_3 + \lambda_1(t)L_1 + \varsigma(\lambda_{max} - \lambda_1(t))\big)e_1 - \left[\frac{e_2^T}{\lambda_1^4(t)} + \frac{e_1^T}{\lambda_1^3(t)}\right](\zeta\varphi(e_1, e_2) - w(t)) \tag{5.47}$$

Finally, let us select $V_3(x) = \frac{1}{2}\zeta^T\zeta$ and find the time derivative, and that is given as $\dot{V}_3(x) = \dot{\zeta}^T\zeta$ combining all the terms for $\dot{V}(x) = \dot{V}_1(x) + \dot{V}_2(x) + \dot{V}_3(x)$

Substitute previous values in the above equation yields

$$\dot{V}(x) \leq -\frac{1}{\lambda_1^4(t)} e_2^T \Big[L_3 + \lambda_1(t) L_4 + \lambda_1(t) L_1 + {\lambda_1}^2(t) + {\lambda_1}^2(t)(L_2 - 1) + {\lambda_1}^4(t) L_2 + 3\varsigma(\lambda_{max} - \lambda_1(t))\Big] e_1 - \frac{1}{\lambda_1^4(t)} e_2^T \left[L_4 + \lambda_1(t)(L_2 - 1) - \frac{{\lambda_1}^4(t)}{2\lambda} + 2\varsigma\left(\frac{\lambda_{max}}{\lambda_1(t)} - 1\right)\right] e_2 - \frac{1}{\lambda_1^3(t)} e_1^T \left[L_3 + \lambda_1(t) L_1 + {\lambda_1}^3(t)\left(L_1 - \frac{\lambda}{2}\right) + \varsigma(\lambda_{max} - \lambda_1(t))\right] e_1 - \zeta\left(\left[\frac{e_2^T}{\lambda_1^4(t)} + \frac{e_1^T}{\lambda_1^3(t)}\right]\varphi(e_1, e_2) - \dot{\zeta}^T\right) + \left[\frac{e_2^T}{\lambda_1^4(t)} + \frac{e_1^T}{\lambda_1^3(t)}\right] w(t) \tag{5.48}$$

The right hand-side of the equation will be negative with proper selection of the correction terms L_1, L_2, L_3, L_4 and $\lambda, \lambda_1(t)$, leaving behind only positive term containing disturbance $w(t)$. Taking the above remarks (5.4)-(5.5) into account, the Lyapunov function guarantees the boundedness of all closed-loop signals and the convergence of tracking error despite the system uncertainties and external force disturbances.

5.6 Simulation Study

The schematic of a two-degree of freedom manipulator is shown in Figure. 5-2. The equation of motion can be written in the form of (5.1) with [49].

$$M_0(q) = \begin{bmatrix} m_1 {l_1}^2 + m_2 \overline{l_2}^2 & m_2 l_2 (l_2 + l_1 \cos\ q_2) \\ m_2 l_2 (l_2 + l_1 \cos\ q_2) & m_2 {l_2}^2 \end{bmatrix} \tag{5.49}$$

$$\mathrm{h}_0(\mathrm{q}, \dot{\mathrm{q}}) = m_2\, l_1\, l_2 \begin{bmatrix} -(2\, \dot{\mathrm{q}}_1\, \dot{\mathrm{q}}_2 + {\dot{\mathrm{q}}_2}^2)\, cos\, q_2 \\ {\dot{\mathrm{q}}_1}^2\, sin\, q_2 \end{bmatrix} + \begin{bmatrix} (m_1 + m_2)\, g\, l_1\, cos\, q_1 + m_2\, g\, l_2\, cos\,(q_1 + q_2) \\ m_2\, g\, l_2\, cos\,(q_1 + q_2) \end{bmatrix} \tag{5.50}$$

In which $\overline{l_2}^2 = l_1{}^2 + l_1{}^2 + 2\, l_1 l_2 \cos\, q_2$, and the Jacobian matrix J is given by:

$$J(q) = \begin{bmatrix} -\,l_1 \sin\, q_1 - l_2 \sin\,(q_1+q_2) & -l_2 \sin\,(q_1+q_2) \\ l_1 \cos\, q_1 + l_2 \cos\,(q_1+q_2) & l_2 \cos\,(q_1+q_2) \end{bmatrix} \tag{5.51}$$

$$\Phi(q\,,\dot{q}\,) = \begin{bmatrix} g l_1 cos q_1 & g l_1 cos q_1 + g l_2 \cos(q_1+q_2) - l_1 l_2 (2\dot{q}_1\dot{q}_2 + \dot{q}_2^2) cos q_2 \\ 0 & l_1 l_2 \dot{q}_1^2 sin q_2 + g l_2 \cos(q_1+q_2) \end{bmatrix} \tag{5.52}$$

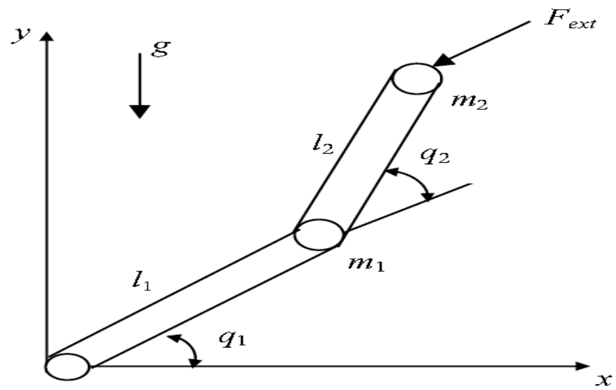


Figure 5-2 Two degree of freedom robot manipulator.

Let us consider the desired trajectory be $q_r(t) = [q_{r1}(t) \quad q_{r2}(t)]^T = 0.5[\sin t \quad \cos t]^T$, and the nominal parameters be $m_{10} = 2$ Kg,$m_{20} = 1$ Kg, $l_1 = l_2 = 0.5$ m and $g = 9.8 \quad m/s^2$.The initial conditions are assumed to be $q(0) = [0.5 \quad 0]^T$and $\dot{q}(0) = [0 \quad 0]^T$. The high gain non-linear observer parameters are assumed to be $L_1 = 1 + \frac{\lambda}{2}, L_2 = 1, L_3 = 50, L_4 = 1 + (\lambda_1)^4$ and $\lambda = \lambda_1$ =100, the robust adaptive controllers are constructed by $K = 2I_{2\times 2}, \Lambda = diag(5{,}8), \delta = 0.2, \sigma = 0.1$. constant parametric uncertainty is considered in dynamical model by taking the mass parameters as $m_1 = m_{10} + \theta_{m_1}$ and $m_2 = m_{20} + \theta_{m_2}$, where $\theta_{m_i}, i = 1{,}2$, denotes and unknown constant assumed to be 20% of the nominal value of the simulation.

In order to verify the effectiveness of the proposed control algorithm, the performance of the closed – loop system is now evaluated here by applying external force, a time varying external force as $F_{ext}(t) = [F_1(t) \quad F_2(t)]^T$ is exerted on end-effector at t=2 sec. The update laws (5.7)-(5.9) are initiated respectively by $\hat{\alpha}(0) = 0.02$ and $\widehat{\Theta} = [0.01 \quad 0.01]^T$. Adaption gains $\gamma_\alpha = 0.03$ and $\Gamma = 0.5I_{2\times2}$.

5.6.1 Simulation Results Related to Constant High Gain Observer Design

In order to study the simulation based on constant high gain observer design with robust adaptive control law, we take up four cases:

Case 1: The proposed scheme is first tested without applying any external disturbance and initial condition.

Case 2: We test the same control law with initial conditions in the absence of external force.

Case 3: Now we apply the external force in this case without any initial condition.

Case 4: Finally we apply initial condition as well as external disturbances to the proposed control and study the performance. Let us now discuss these cases in detail.

Case 1: The performance of the closed loop system is evaluated here without applying and initial condition and external force.

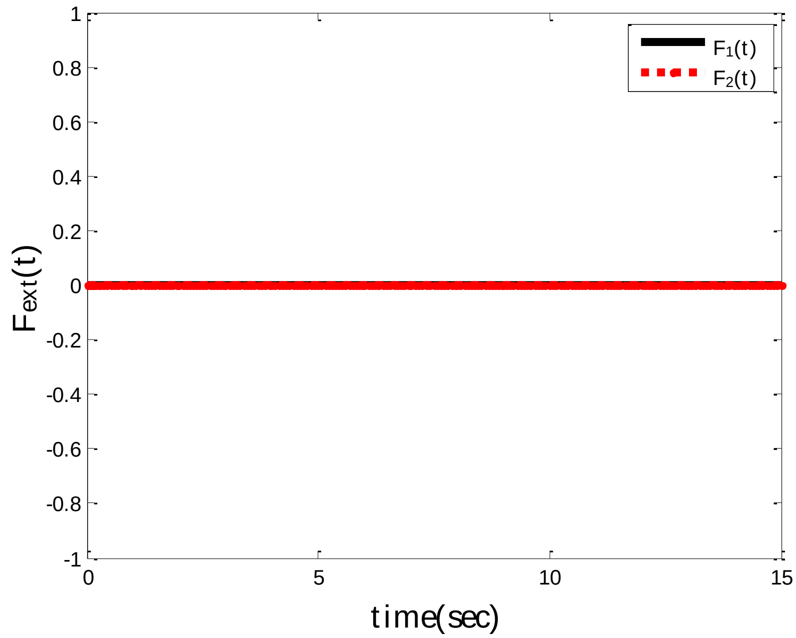


Figure 5-3 Time response of external force disturbances for case 1.

This graphs shows that, there is no external force applied during the simulation.

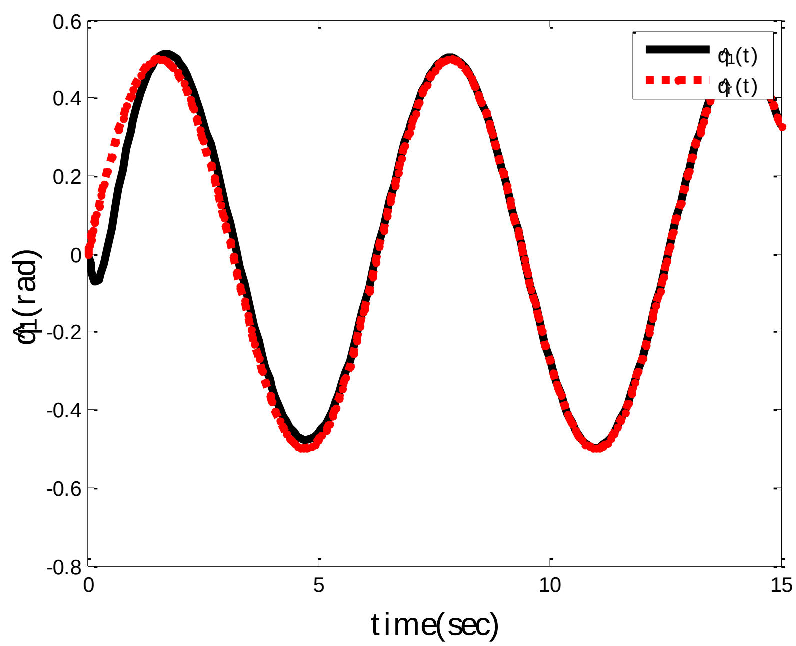


Figure 5-4 Time response of the desired trajectory to the estimated angular position 1 of the robot manipulator for case1.

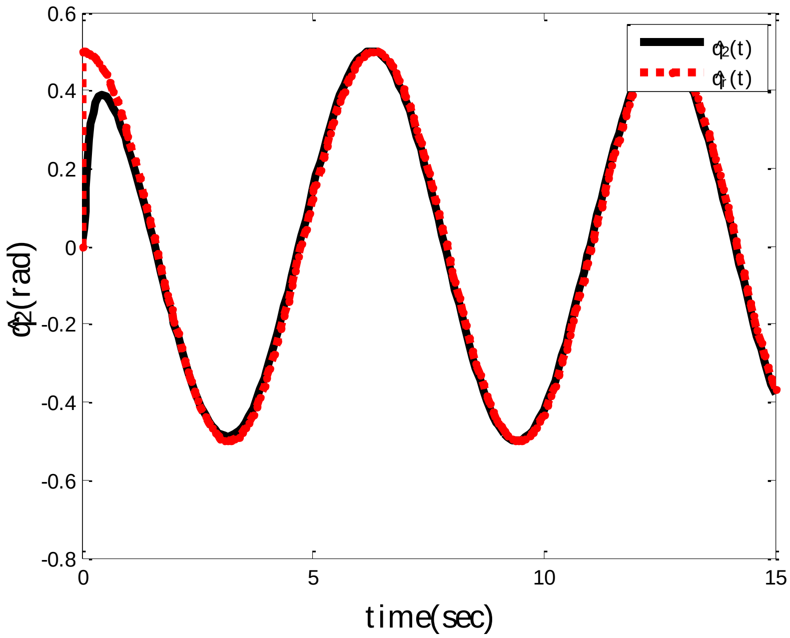


Figure 5-5 Time response of the desired trajectory to the estimated angular position 2 of the robot manipulator for case1.

In this graph, the time response of the angular position, of the robot manipulator is plotted, it can be clearly seen that the angular position $\hat{q}_1(t), \hat{q}_2(t)$ of the manipulator is able to track the desired trajectory $q_r(t)$ and also both the responses are starting from the same point, it because of zero initial condition.

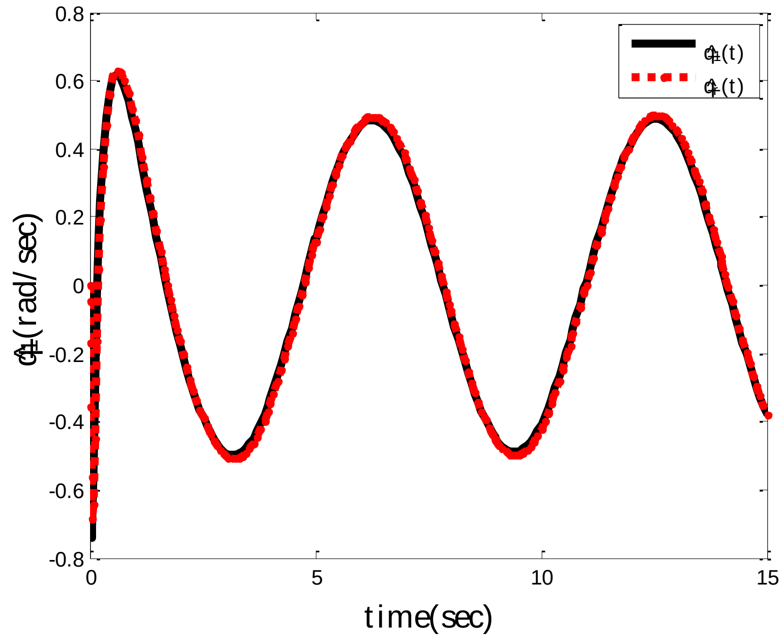


Figure 5-6 Time response of the desired trajectory and the derivative of estimated response angular velocity 1 of the robot manipulator for case 1.

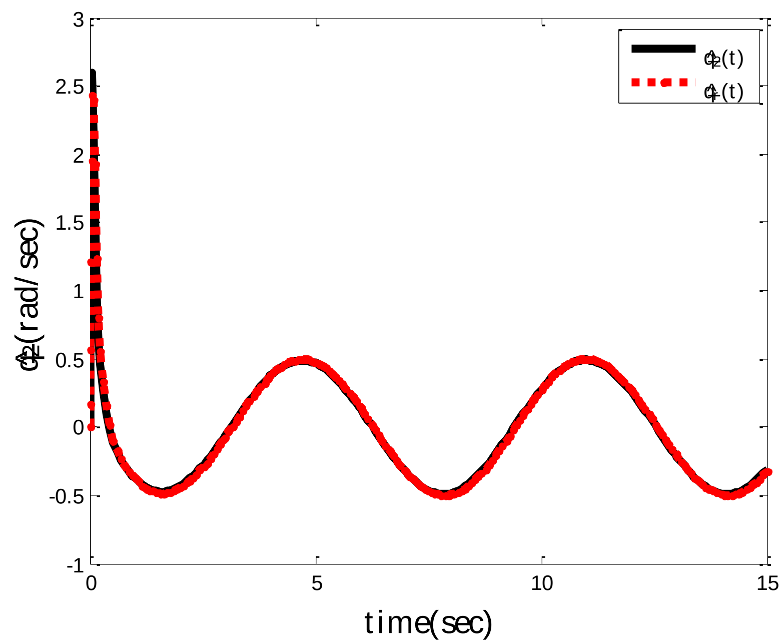


Figure 5-7 Time response of the desired trajectory and the derivative of estimated response angular velocity 2 of the robot manipulator for case 1.

The angular velocity $\dot{\hat{q}}_1(t)$, $\dot{\hat{q}}_2(t)$ of the robot manipulator is drawn with respect to the derivative of the desired trajectory $\dot{q}_r(t)$, the graphs shows the perfect tracking is achieved.

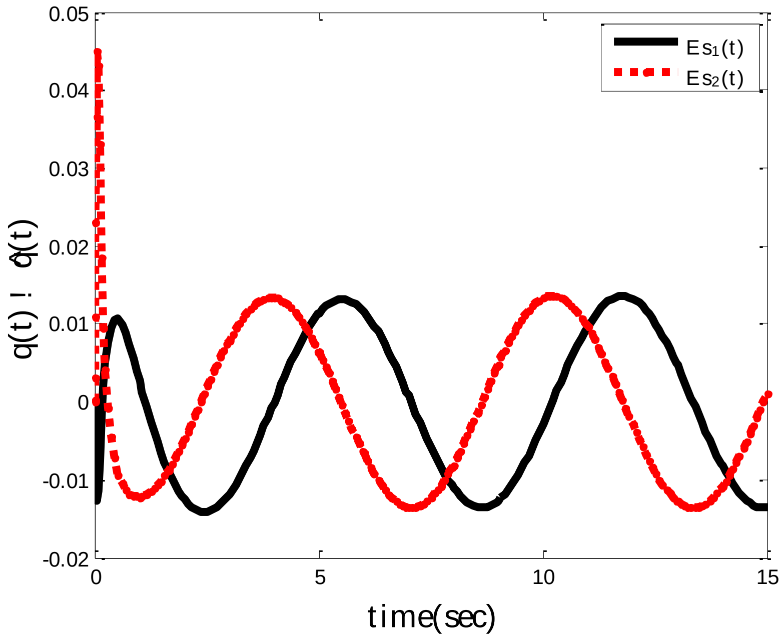


Figure 5-8 Convergence of position tracking error of the robot manipulator for case 1.

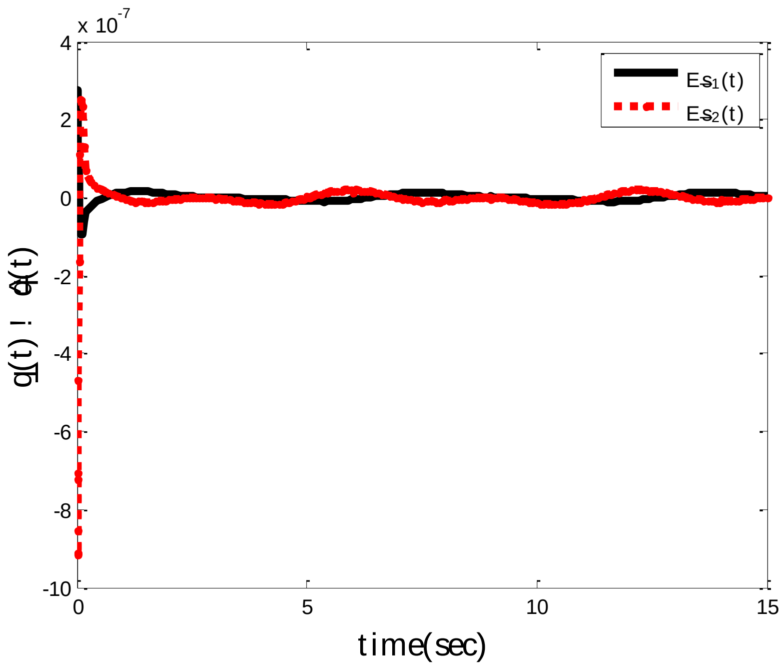


Figure 5-9 Convergence of velocity tracking error of the robot manipulator for case 1.

These graphs shows the controller's convergent property, convergence of position tracking error which is the difference between the actual position of the robot manipulator with respect to the estimated position of the robot manipulator $q - \hat{q}$ the error as we can see, between them is very small. The convergence of velocity tracking error which is the difference the actual velocity of the manipulator with respect to the estimated velocity of the manipulator $\dot{q} - \dot{\hat{q}}$ is of the order 10^{-7} as considered as negligible. And it is also a fact that, smaller the error, better the result and in our case error is very small so the performance of the manipulator will increase considerably.

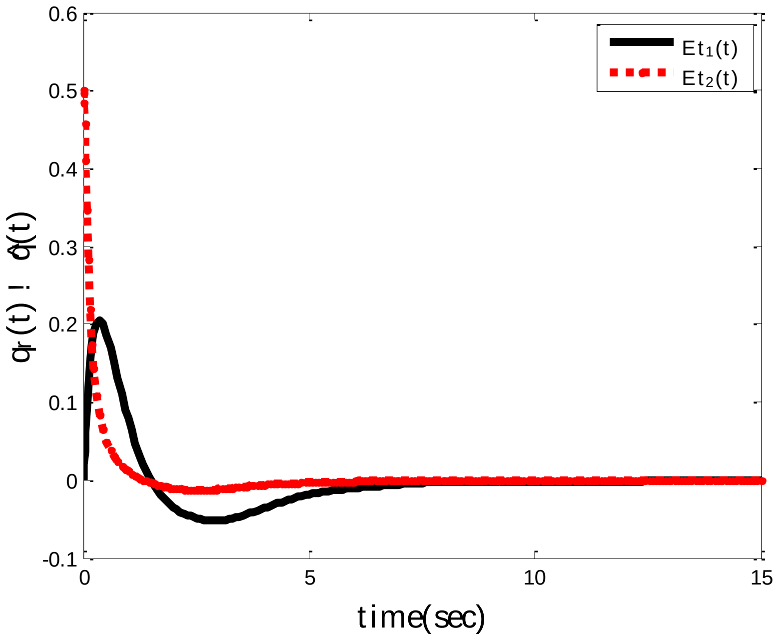


Figure 5-10 Robust Convergence of position tracking error of the robot manipulator for case 1.

These are results when estimated angular position and angular velocity of the robot manipulator are plotted with respect to reference signal and derivative of reference signal respectively.

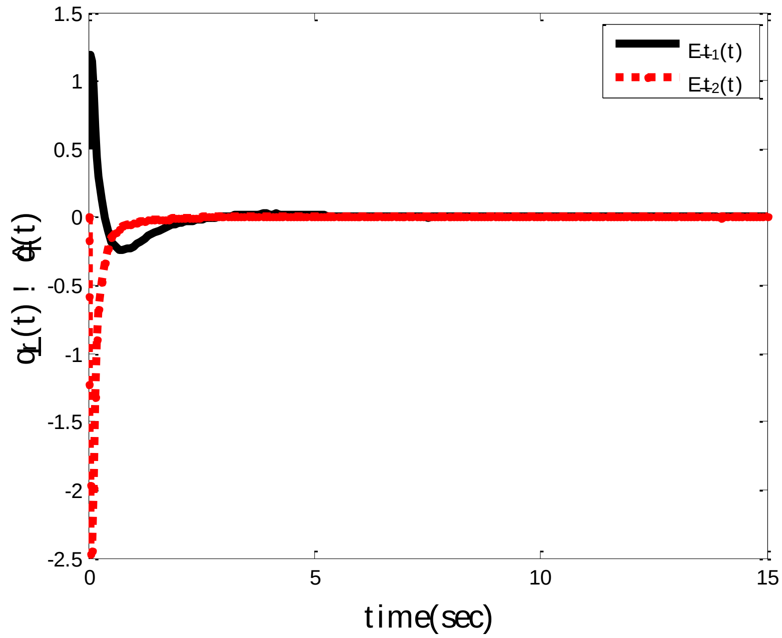


Figure 5-11 Robust Convergence of velocity tracking error of the robot manipulator for case 1.

This graphs validates the robust convergence phenomenon of the systems effectively.

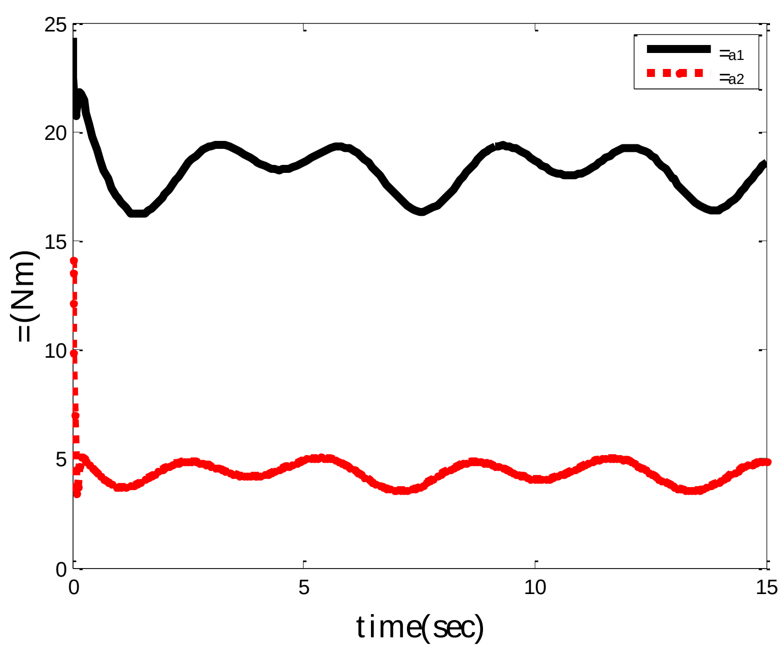


Figure 5-12 Applied input torque for case 1.

The control input or the control effort required by the robot manipulator is shown in the above graph. Here we can see there are no abrupt changes in the effort required, the waveform is smooth and moreover slightly oscillating between the upper and lower control limit.

Case 2: The performance of the closed loop system is evaluated here by applying initial condition and no external force.

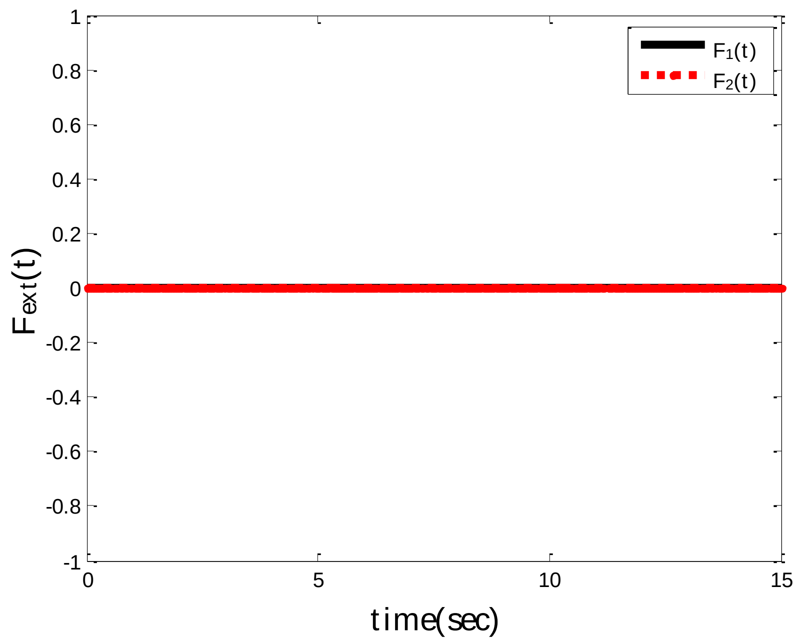


Figure 5-13 Time response of external force disturbances for case 2.

The graph shows no external disturbance because in this case we study the behavior of the robot manipulator just by applying initial condition.

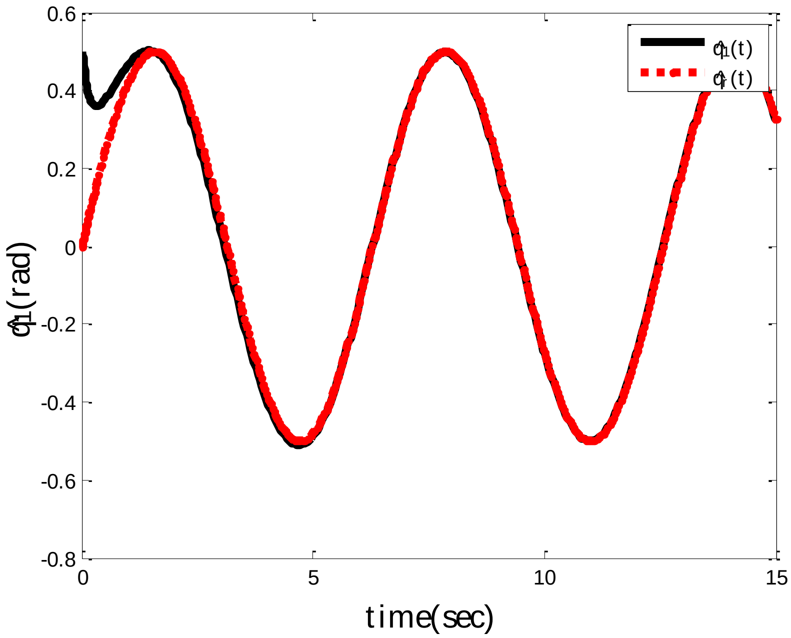


Figure 5-14 Time response of the desired trajectory to the estimated angular position 1 of the robot manipulator for case 2.

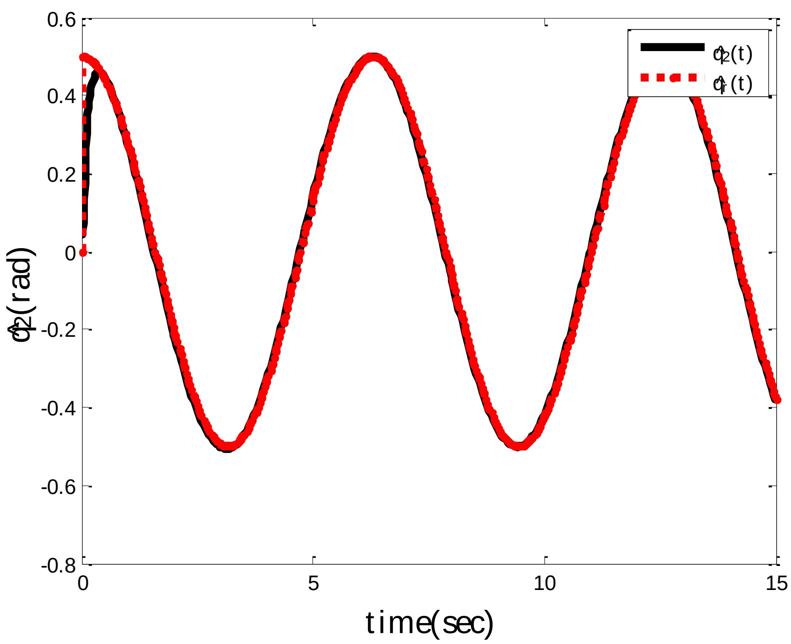


Figure 5-15 Time response of the desired trajectory to the estimated angular position 2 of the robot manipulator for case 2.

The graph shows the angular position of the robot manipulator $\hat{q}_1(t), \hat{q}_2(t)$ plotted with respect to desired trajectory. Here we can see that the robot manipulator is able to track the desired signal. The waveforms for estimated angular position are starting from 0.5 because of the implementation of the initial inside the system.

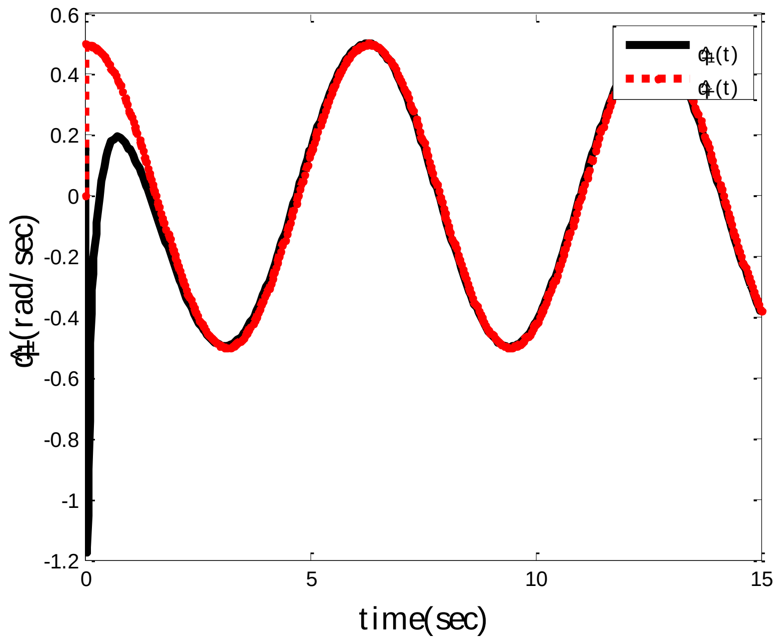


Figure 5-16 Time response of the desired trajectory and the derivative of estimated angular velocity 1 of the robot manipulator for case 2.

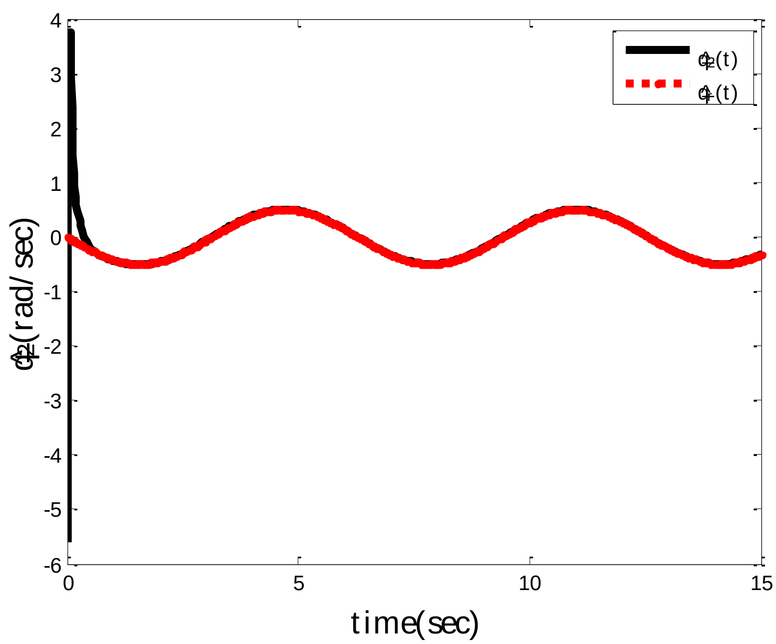


Figure 5-17 Time response of the desired trajectory and the derivative of estimated angular velocity 2 of the robot manipulator for case 2.

Because of the change in the initial conditions, we can see the variations at the start in the above waveforms corresponding to the angular velocity $\dot{\hat{q}}_1(t)$, $\dot{\hat{q}}_2(t)$ of the robot manipulator. Also, it is clear that with the change in initial conditions the manipulator is still able to track the desired trajectory.

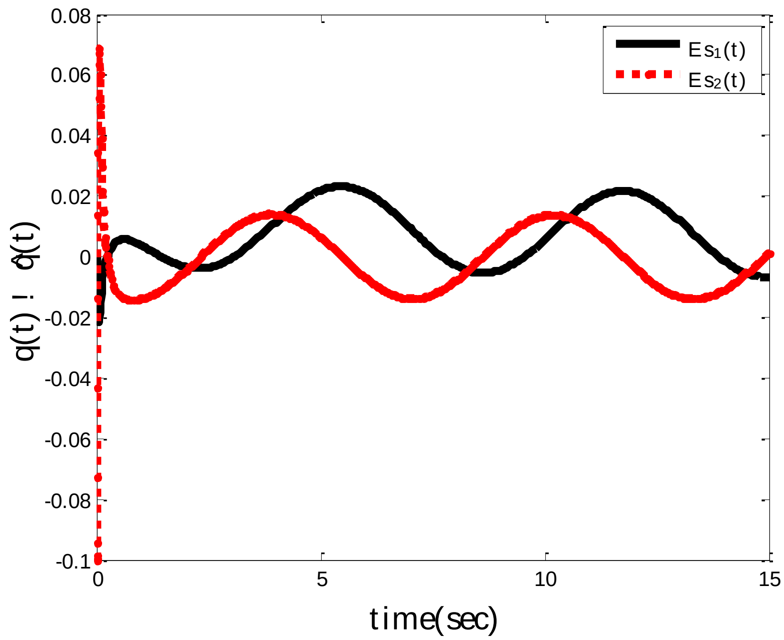


Figure 5-18 Convergence of position tracking error of the robot manipulator for case 2.

Form the above graphs it is clear that the convergence of position $q - \hat{q}$ tracking error is achieved, the error as seen from the graphs are small and close to zero.

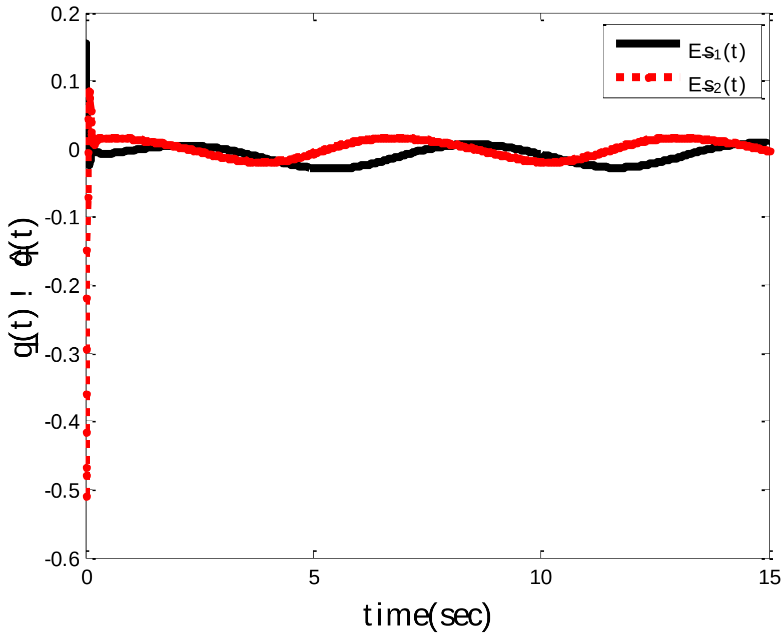


Figure 5-19 Convergence of velocity tracking error of the robot manipulator for case 2.

Form the above graphs it is clear that the convergence of position $q - \hat{q}$ and velocity $\dot{q} - \dot{\hat{q}}$ tracking error is achieved, the error as seen from the graphs are small and close to zero.

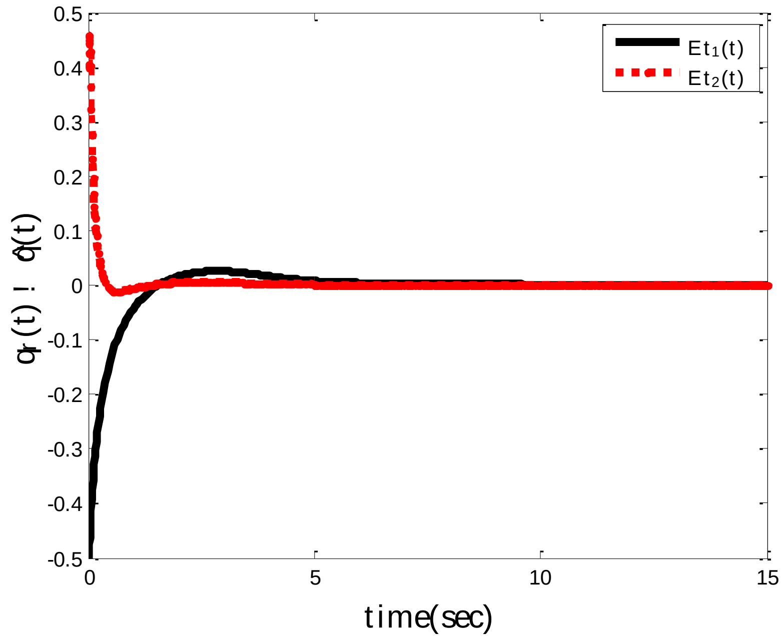


Figure 5-20 Robust Convergence of position tracking error of the robot manipulator for case 2.

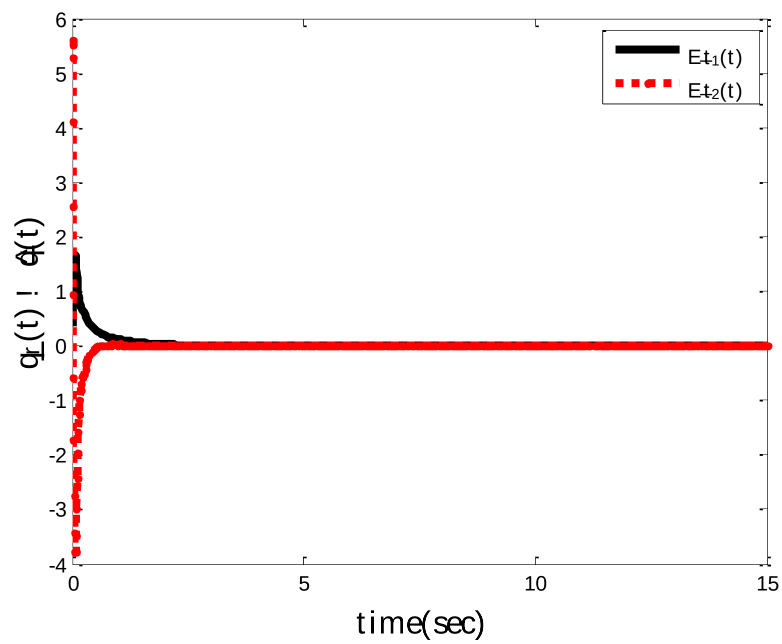


Figure 5-21 Robust Convergence of velocity tracking error of the robot manipulator for case 2.

Here graphs are plotted between estimated angular position and estimated angular velocity with respect to desired trajectory. The error difference them after a short duration dies down to zero, concluding the robust convergence property of the manipulator.

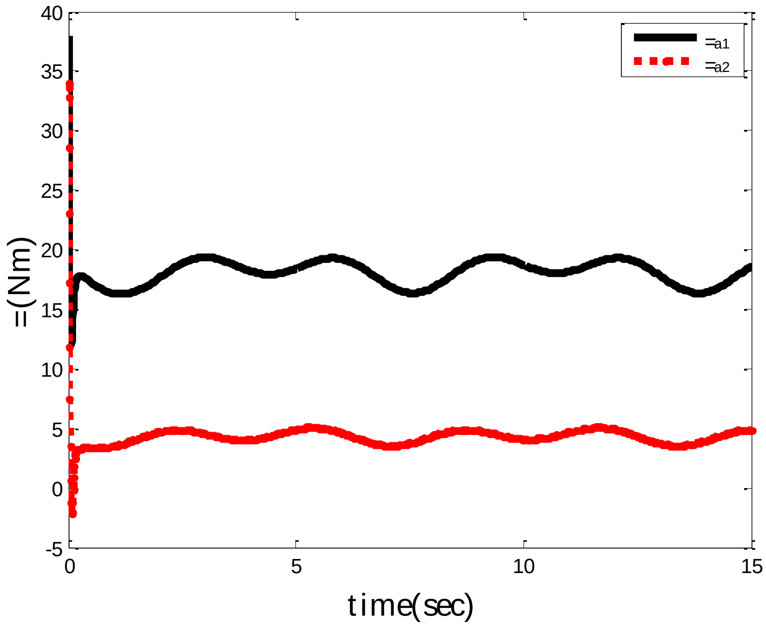


Figure 5-22 Applied input torques for case 2.

With the addition of the initial condition, there is no change the applied input torque of the manipulator, the wave is smooth without any chattering effect. Also, we can see that the control inputs $\tau_{a1}(t)(-)\ and\ \tau_{a2}(t)(\cdots)$ are bounded and slightly oscillating within that bound.

Case 3: The performance of the closed loop system is evaluated here by applying external force without any initial condition.

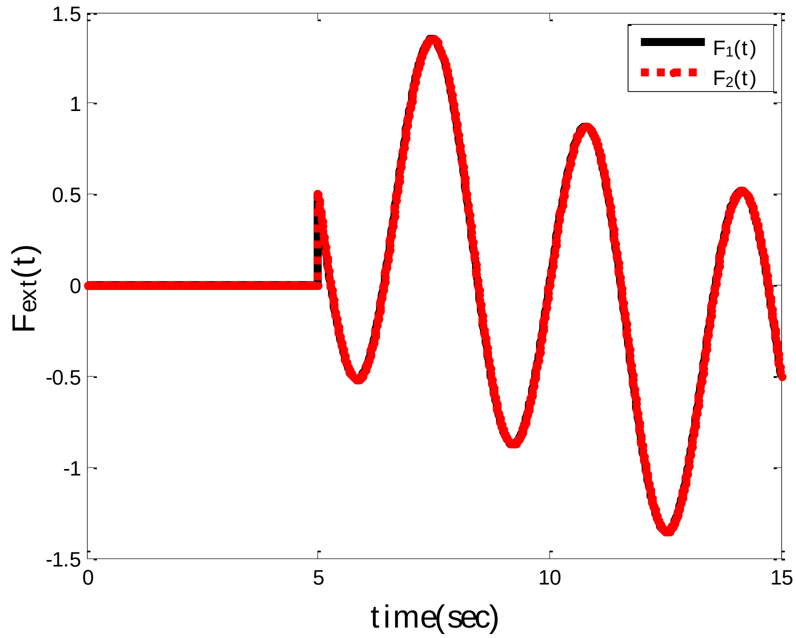


Figure 5-23 Time response of external force disturbances for case 3.

In this case we are adding external disturbance to the system and this disturbance is added at t=5 sec, this can be clearly seen from the above figure.

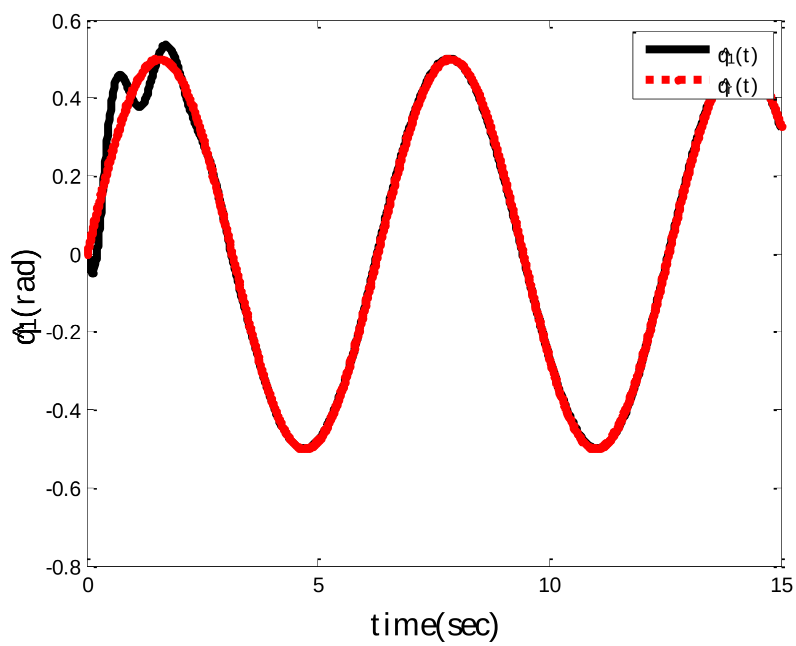


Figure 5-24 Time response of the desired trajectory to the estimated angular position 1 of the robot manipulator for case 3.

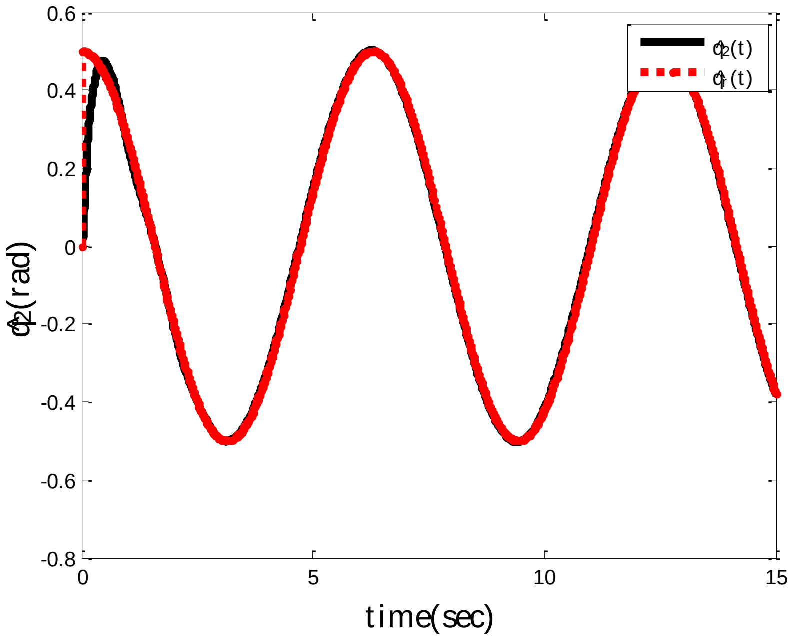


Figure 5-25 Time response of the desired trajectory to the estimated angular position 2 of the robot manipulator for case 3.

This graph explains the concept of trajectory tracking of the robot manipulator efficiently, in the above figures we can see that the time response of the desired trajectory to the estimated angular position of the robot manipulator is a perfect tracking phenomenon.

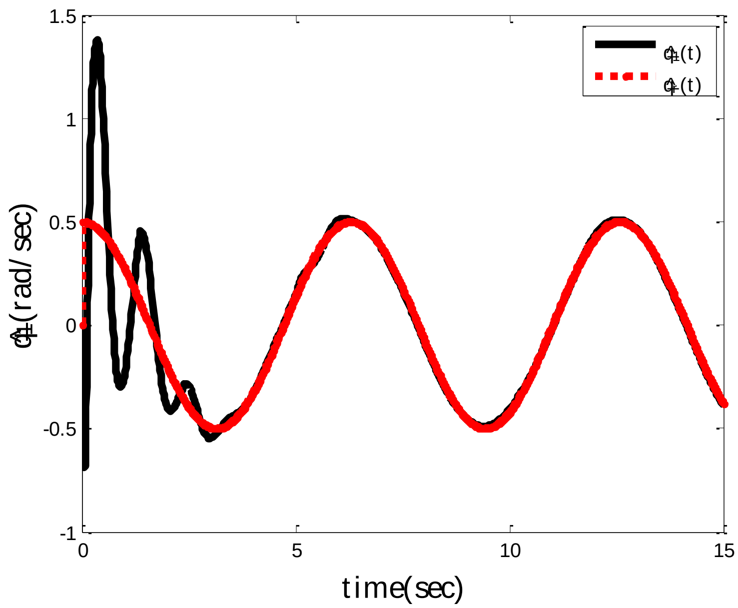


Figure 5-26 Time response of the desired trajectory and the derivative of estimated angular velocity 1 of the robot manipulator for case 3.

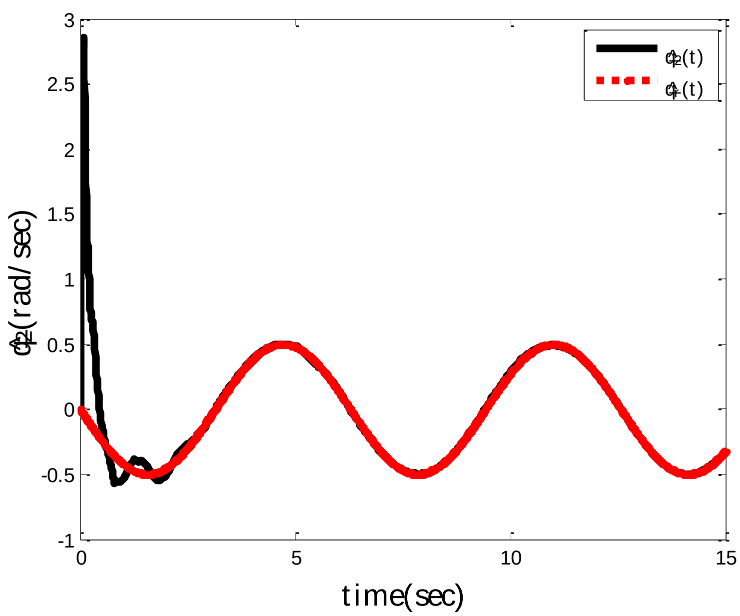


Figure 5-27 Time response of the desired trajectory and the derivative of estimated angular velocity 2 of the robot manipulator for case 3.

Time response of the desired trajectory $\dot{q}_r(t)$ with respect to the estimated response $\dot{\hat{q}}_1(t)$, $\dot{\hat{q}}_2(t)$ which is the angular velocity is plotted here and the robot manipulator is able to exhibit efficient tracking performance. The oscillations at the start is the time required by the manipulator to track the desired signal.

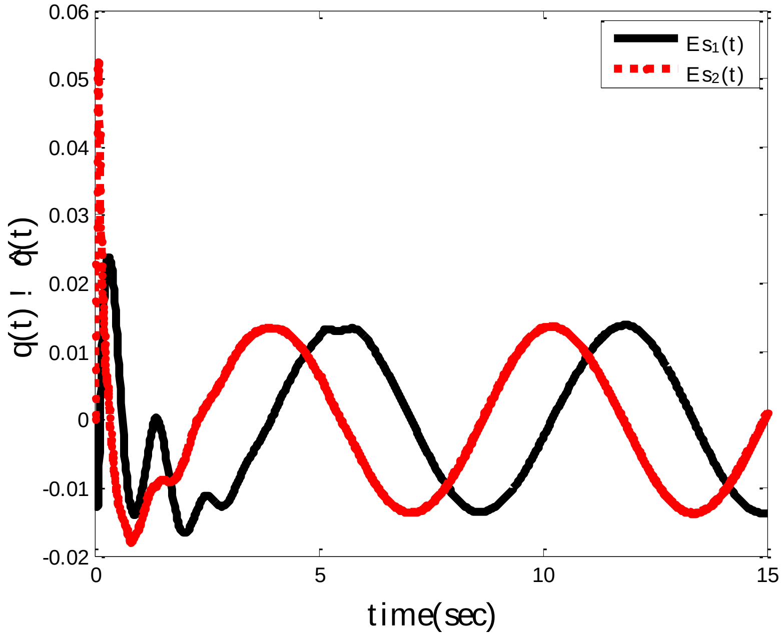


Figure 5-28 Convergence of position tracking error of the robot manipulator for case 3.

This graph explains the convergence of position tracking error phenomenon of the robot manipulator. Position tracking error is determined by taking the difference between the actual position of the manipulator to the estimated position of the manipulator. Here we can see that the tracking error is small and bounded close to zero.

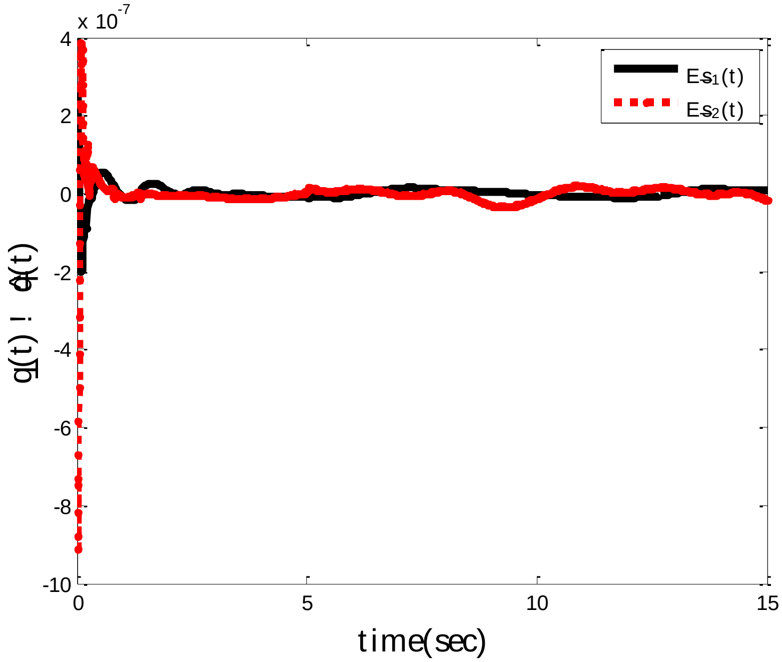


Figure 5-29 Convergence of velocity tracking error of the robot manipulator for case 3.

Here we can see that the velocity tracking error is very small and bounded close to zero.

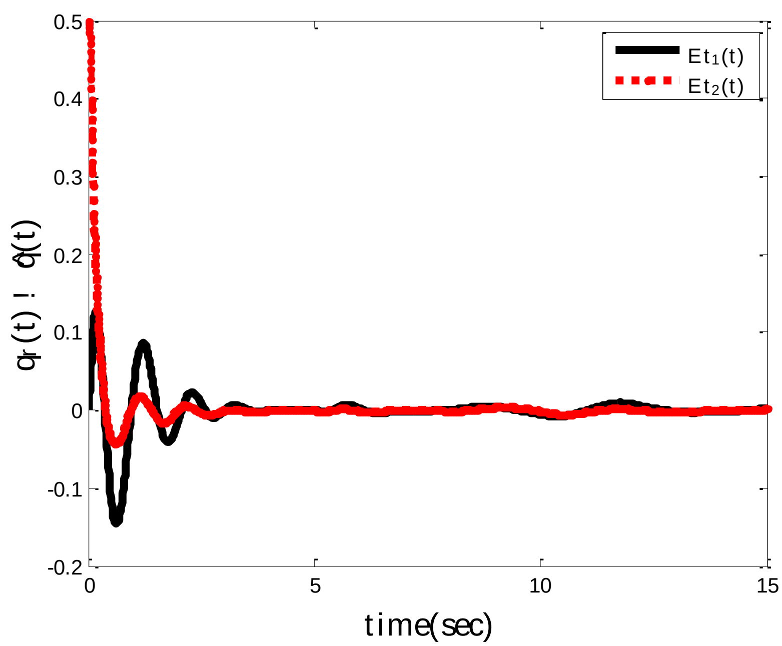


Figure 5-30 Robust Convergence of position tracking error of the robot manipulator for case 3.

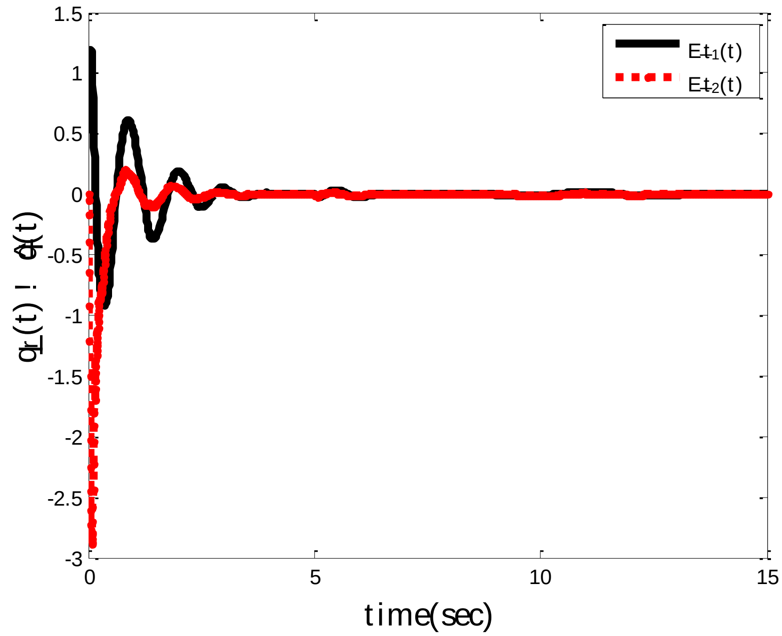


Figure 5-31 Robust Convergence of velocity tracking error of the robot manipulator for case 3.

To explain the concept of robust convergence phenomenon of the robot manipulator the above graphs are plotted. The first figure above is the difference between the desired position to the estimated position of the robot manipulator and it clearly shows that the error decays to zero, same is the case when we plot the graph between the desired velocity to the actual velocity of the manipulator, as this can be seen from the second figure shown above.

The control effort or the applied input torque required by the system is shown below, the graph can be seen with variations because of the implementation of external force and after a short duration they tend to oscillate within a control bound.

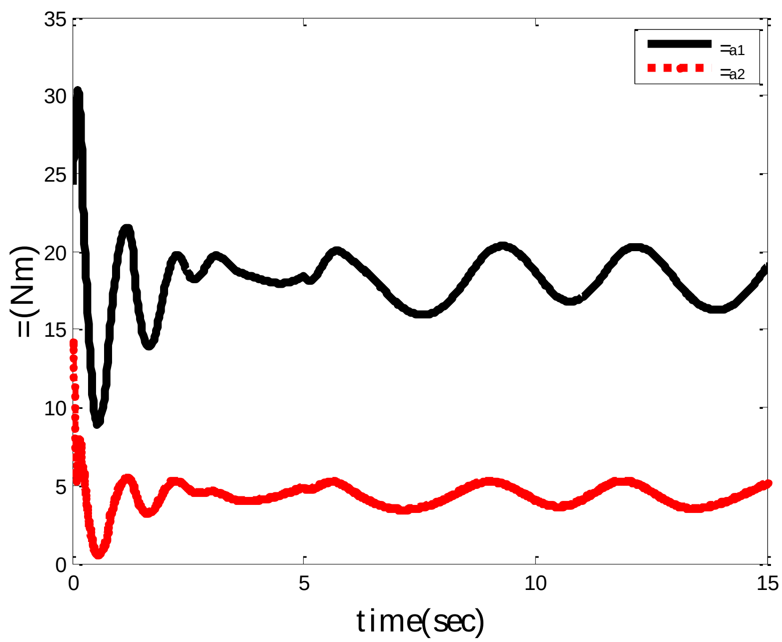


Figure 5-32 Applied input torques for case 3.

Case 4: The performance of the closed loop system is evaluated here by applying external force and initial condition.

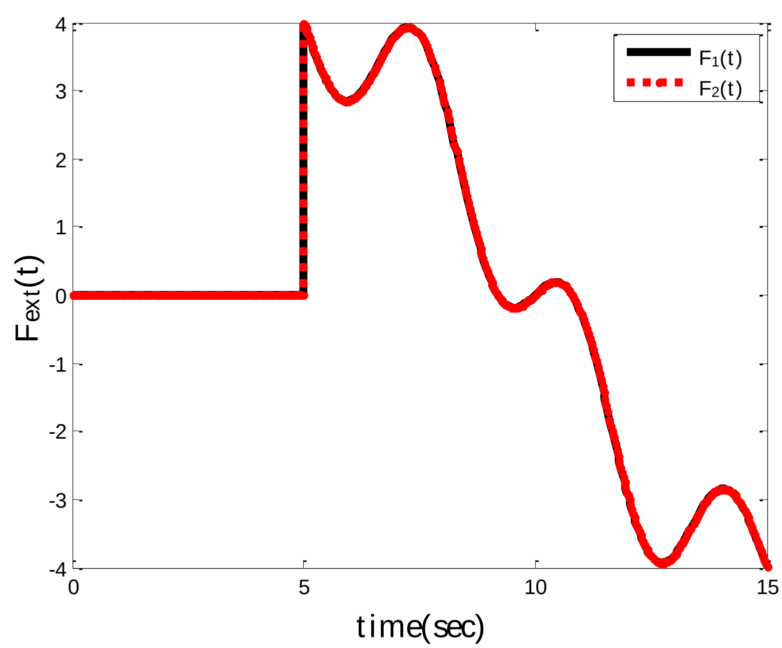


Figure 5-33 Time response of external force disturbances for case 4.

The applied external force disturbance can be seen above, the force is applied after t=5sec.

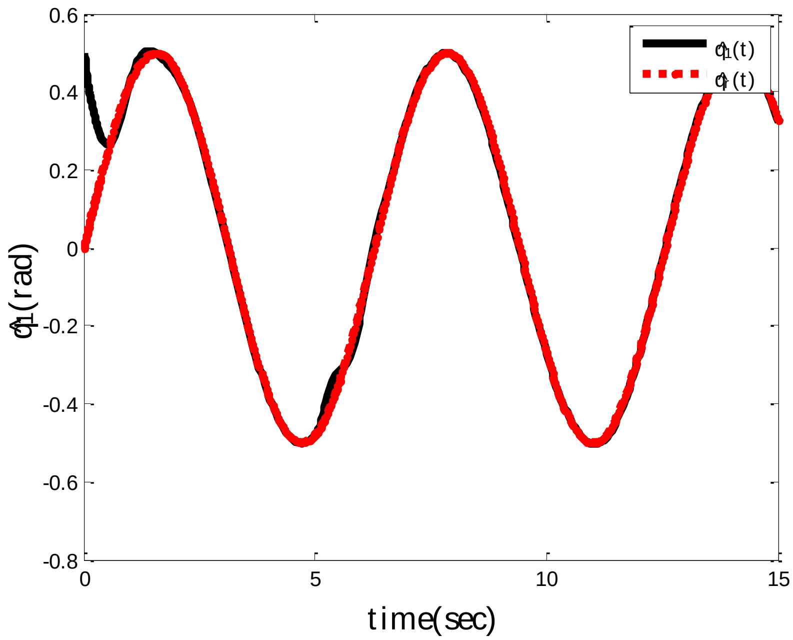


Figure 5-34 Time response of the desired trajectory to the estimated angular position 1 of the robot manipulator for case 4.

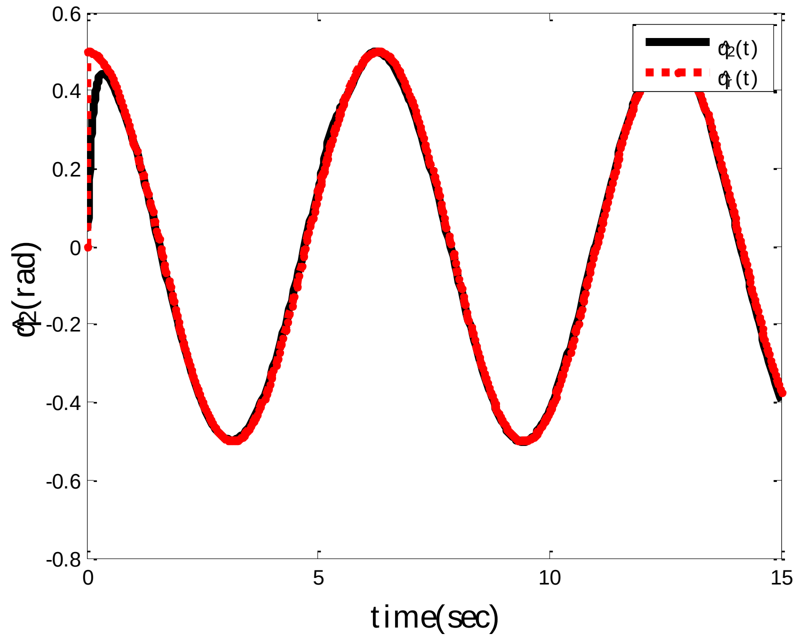


Figure 5-35 Time response of the desired trajectory to the estimated angular position 2 of the robot manipulator for case 4.

In order to validate the control law, these graph are plotted which shows the perfect tracking of the angular position $\hat{q}(t)$ of the robot manipulator with the desired behavior $q_r(t)$, slight variation at t=5 sec is due to implementation the disturbance signal. The waveforms have different starting point due change in the initial condition.

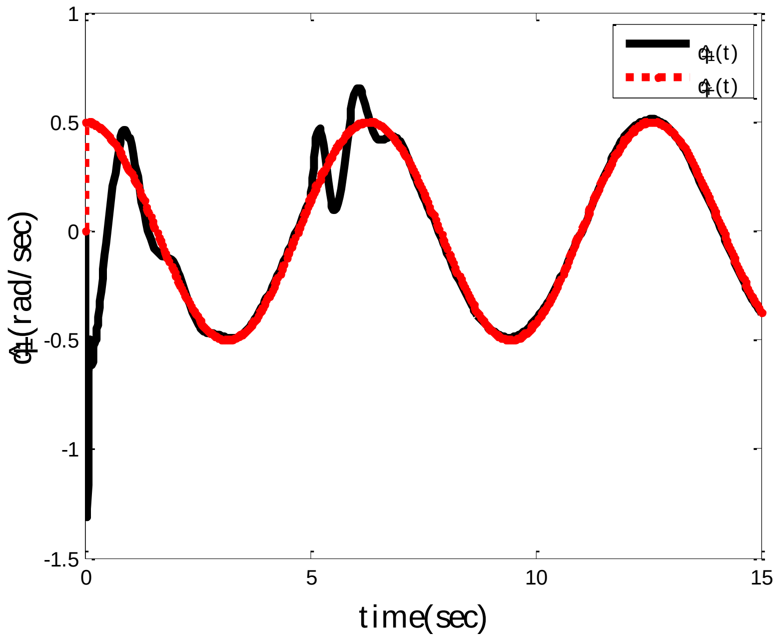


Figure 5-36 Time response of the desired trajectory and the derivative of estimated response angular velocity 1 of the robot manipulator for case 4.

The above figure corresponds to the angular velocity of the manipulator plotted with respect to the desired trajectory. Due to the external disturbance we see variations in the waveforms but within small duration the manipulator is again able to track the desired trajectory.

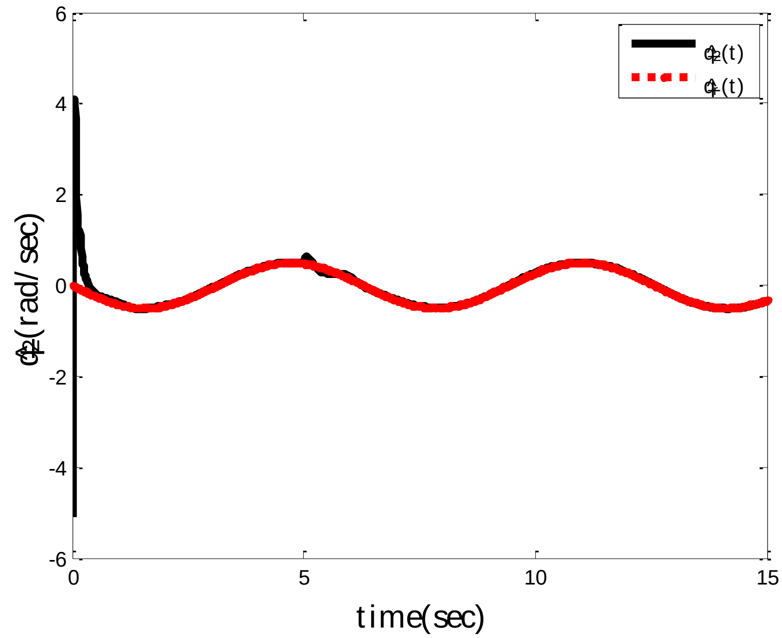


Figure 5-37 Time response of the desired trajectory and the derivative of estimated response angular velocity 2 of the robot manipulator for case 4.

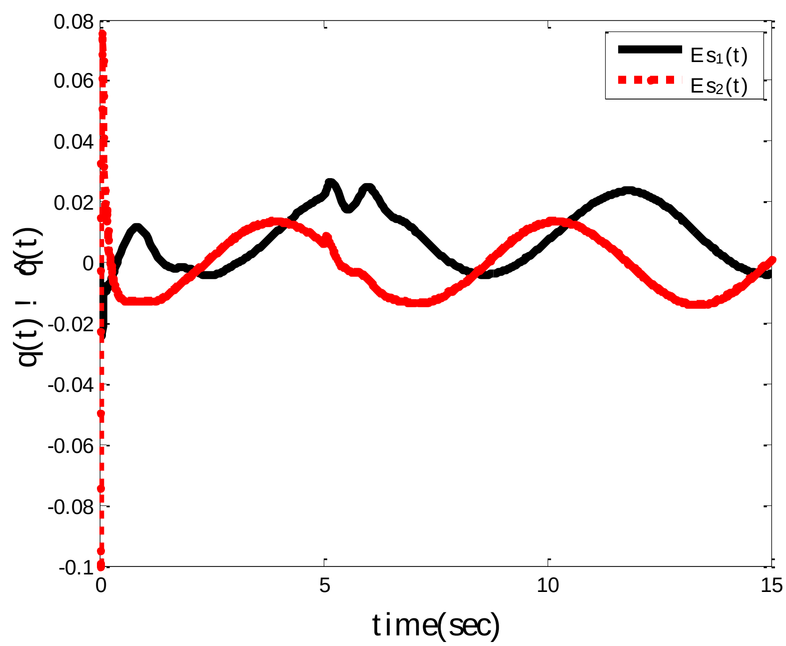


Figure 5-38 Convergence of position tracking error of the robot manipulator for case 4.

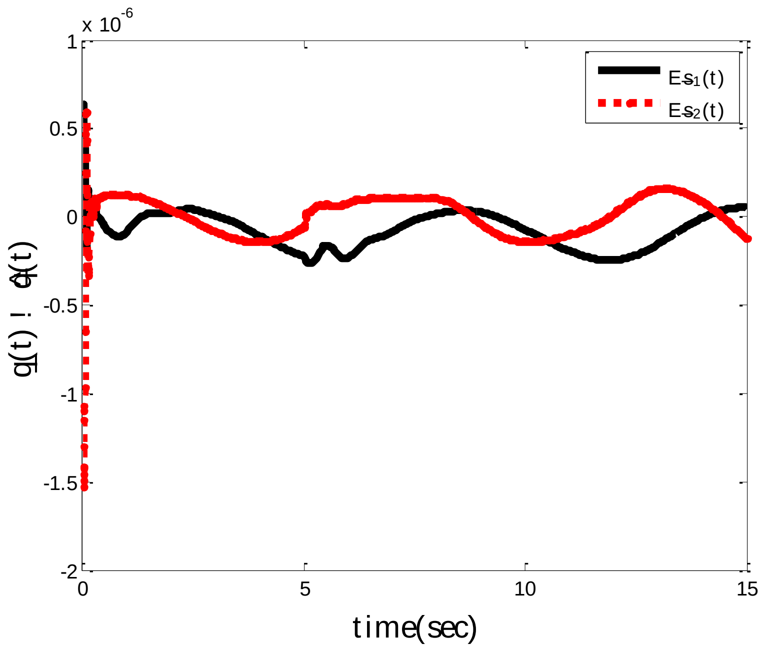


Figure 5-39 Convergence of velocity tracking error of the robot manipulator for case 4.

The difference between the actual position to the estimated position called as position tracking error is plotted showing convergence phenomenon. This result is same when angular velocity tracking error is plotted.

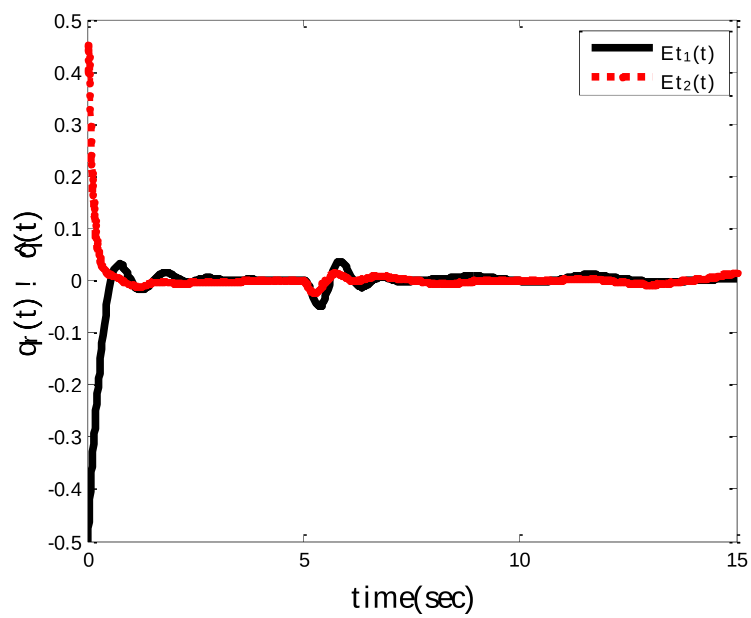


Figure 5-40 Robust Convergence of position tracking error of the robot manipulator for case 4.

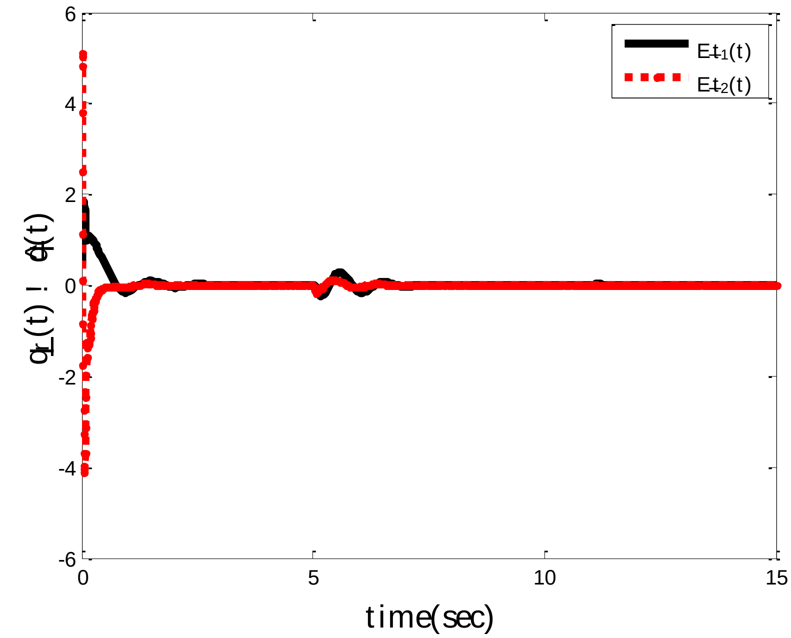


Figure 5-41 Robust Convergence of velocity tracking error of the robot manipulator for case 4.

The robust convergence of position and velocity tracking error of the robot manipulator is shown in the above figure.

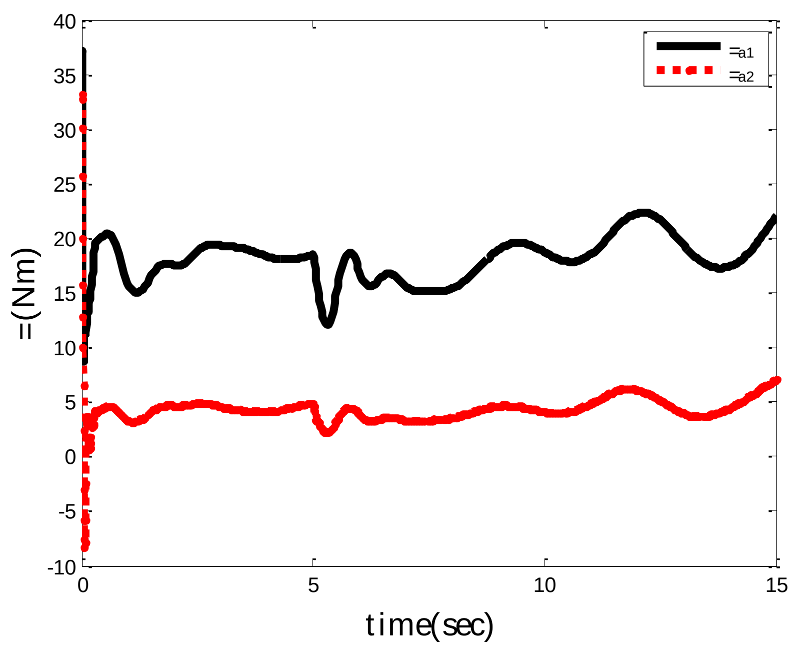


Figure 5-42 Applied input torques for case 4.

The required control effort also known as applied input torque $\tau_{a1}(t)(-)\ and\ \tau_{a2}(t)(\cdots)$ shows a smooth behavior.

5.6.2 Simulation Results Related to Time varying High Gain Observer Design

In order to study the simulation based on time varying high gain observer design with robust adaptive control law, we take up four cases:

Case 1: The proposed scheme is first tested without applying any external disturbance and initial condition.

Case 2: We test the same control law with initial conditions in the absence of external force.

Case 3: Now we apply the external force in this case without any initial condition.

Case 4: Finally we apply initial condition as well as external disturbances to the proposed control and study the performance. Let us now discuss these cases in detail.

Case 1: The performance of the closed loop system is evaluated here without applying and initial condition and external force.

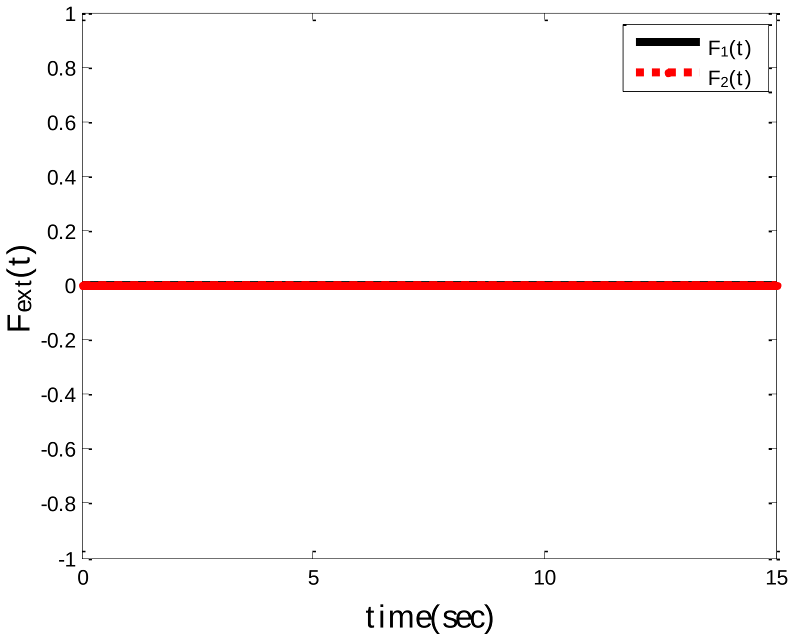


Figure 5-43 Time response of external force disturbances for case 1.

This graphs shows that, there is no external force applied during the simulation.

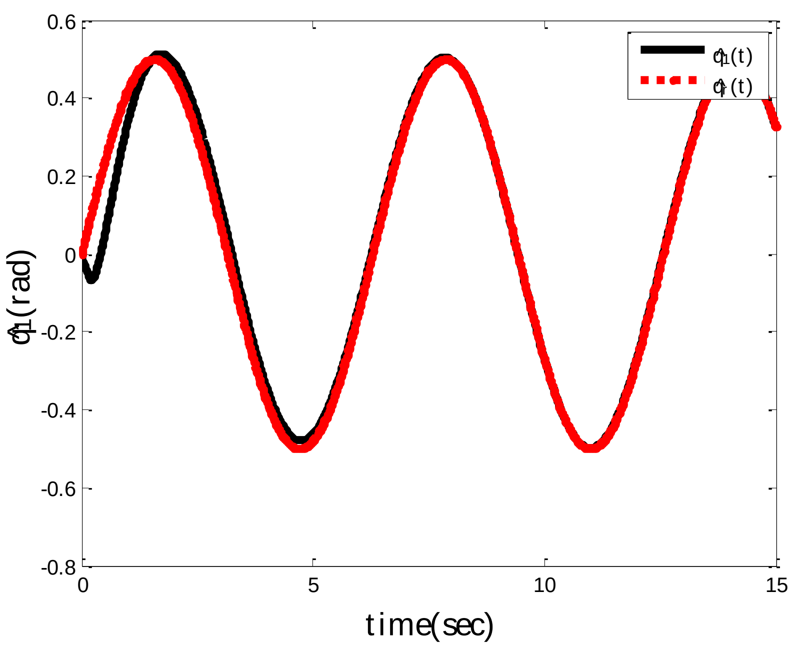


Figure 5-44 Time response of the desired trajectory to the estimated angular position 1 of the robot manipulator for case1.

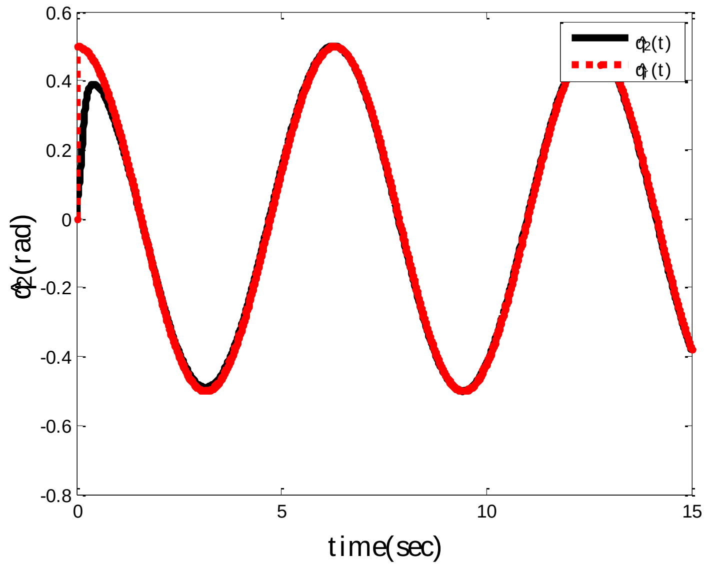


Figure 5-45 Time response of the desired trajectory to the estimated angular position 2 of the robot manipulator for case1.

In this graph, the time response of the angular position, of the robot manipulator is plotted, it can be clearly seen that the angular position $\hat{q}_1(t), \hat{q}_2(t)$ of the manipulator is able to track the desired trajectory $q_r(t)$ and also both the responses are starting from the same point, it because of zero initial condition.

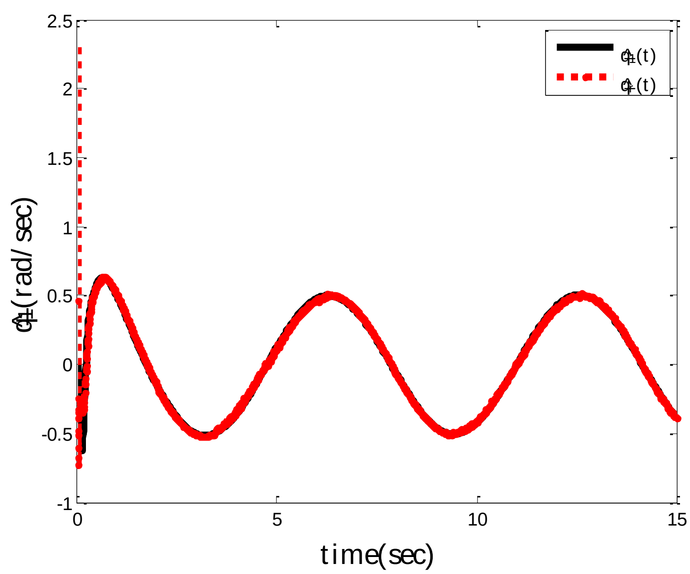


Figure 5-46 Time response of the desired trajectory and the derivative of estimated response angular velocity 1 of the robot manipulator for case 1.

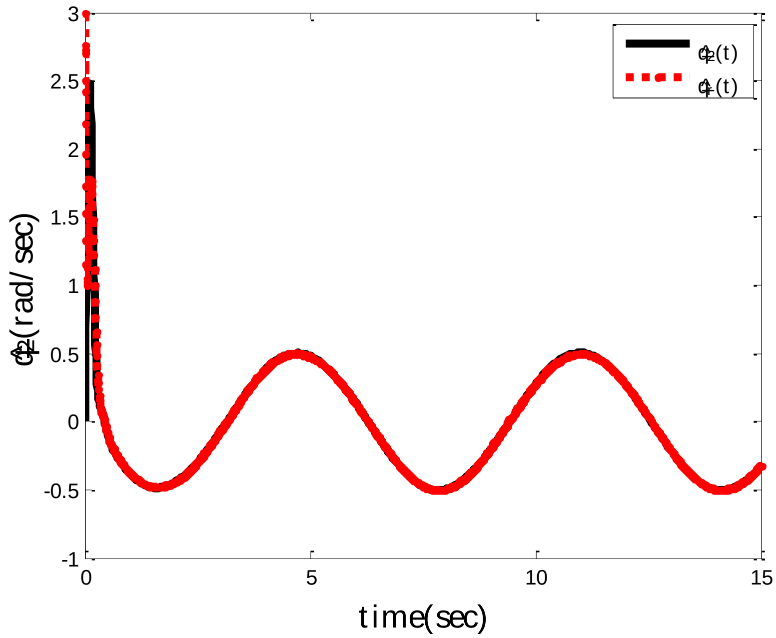


Figure 5-47 Time response of the desired trajectory and the derivative of estimated response angular velocity 2 of the robot manipulator for case 1.

The angular velocity $\dot{\hat{q}}_1(t)$, $\dot{\hat{q}}_2(t)$ of the robot manipulator is drawn with respect to the derivative of the desired trajectory $\dot{q}_r(t)$, the graphs shows the perfect tracking is achieved.

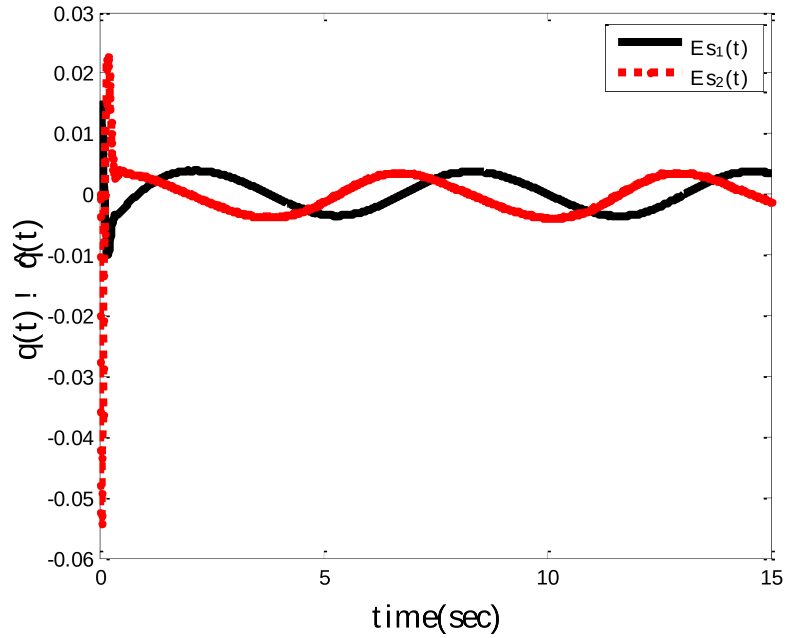


Figure 5-48 Convergence of position tracking error of the robot manipulator for case 1.

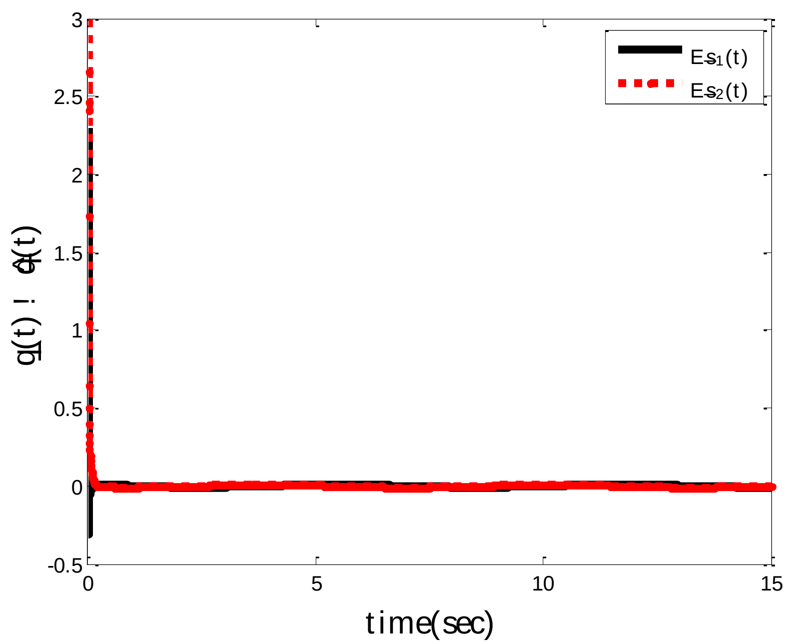


Figure 5-49 Convergence of velocity tracking error of the robot manipulator for case 1.

These graphs shows the controller's convergent property, convergence of position tracking error which is the difference between the actual position of the robot manipulator with respect to the estimated position of the robot manipulator $q - \hat{q}$ the error as we can see, between them is very small. The convergence of velocity tracking error which is the difference the actual velocity of the manipulator with respect to the estimated velocity of the manipulator $\dot{q} - \dot{\hat{q}}$ is of the order 10^{-7} as considered as negligible. And it is also a fact that, smaller the error, better the result and in our case error is very small so the performance of the manipulator will increase considerably.

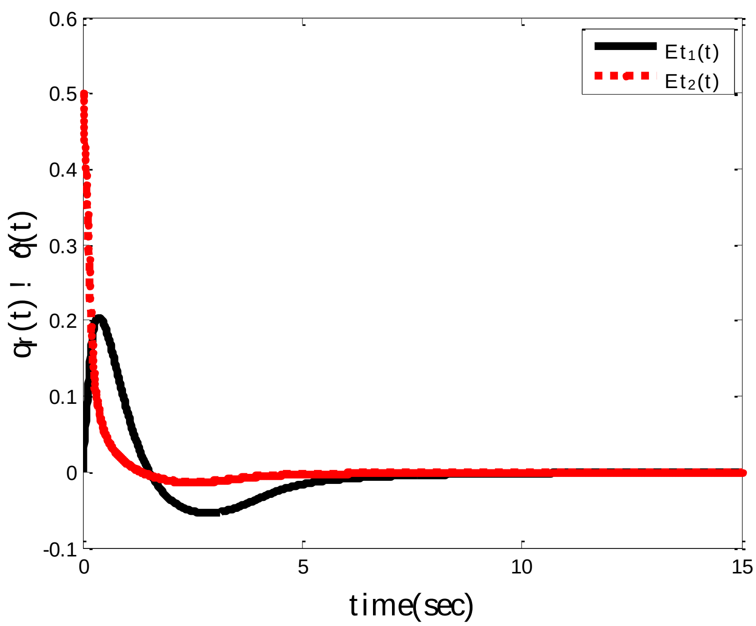


Figure 5-50 Robust Convergence of position tracking error of the robot manipulator for case 1.

These are results when estimated angular position and angular velocity of the robot manipulator are plotted with respect to reference signal and derivative of reference signal respectively.

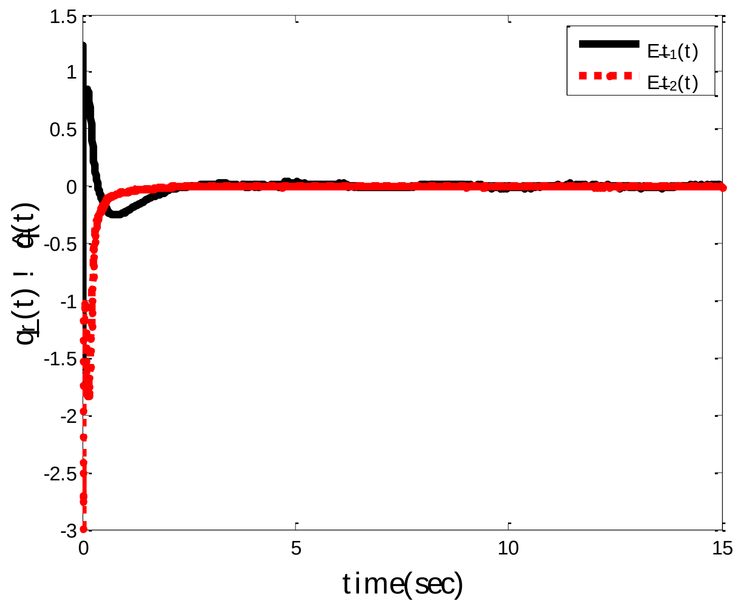


Figure 5-51 Robust Convergence of velocity tracking error of the robot manipulator for case 1.

This graphs validates the robust convergence phenomenon of the systems effectively.

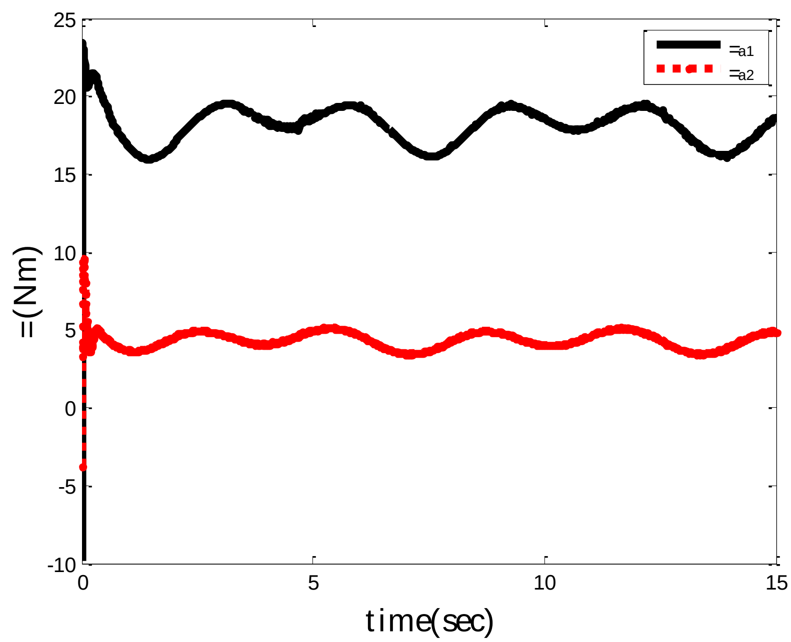


Figure 5-52 Applied input torque for case 1.

The control input or the control effort required by the robot manipulator is shown in the above graph. Here we can see there are no abrupt changes in the effort required, the waveform is smooth and moreover slightly oscillating between the upper and lower control limit.

Case 2: The performance of the closed loop system is evaluated here by applying initial condition and no external force.

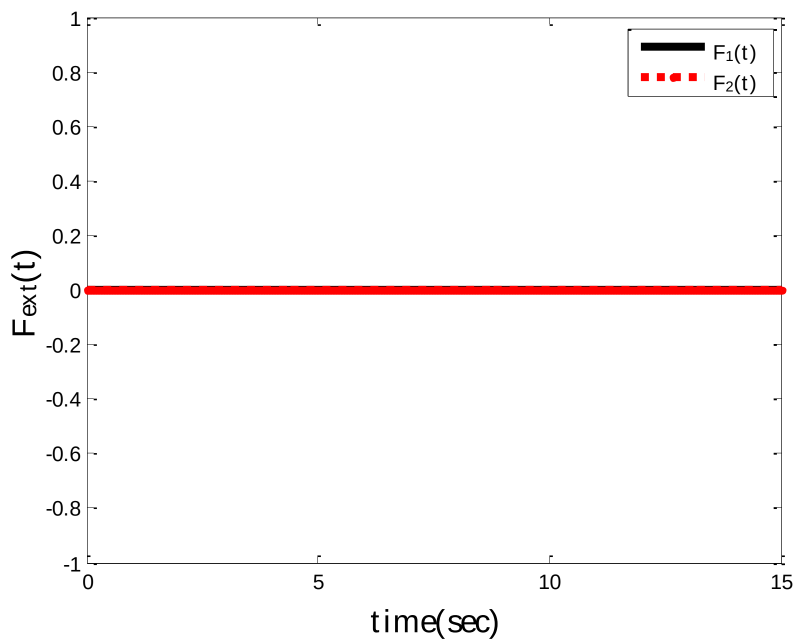


Figure 5-53 Time response of external force disturbances for case 2.

The graph shows no external disturbance because in this case we study the behavior of the robot manipulator just by applying initial condition.

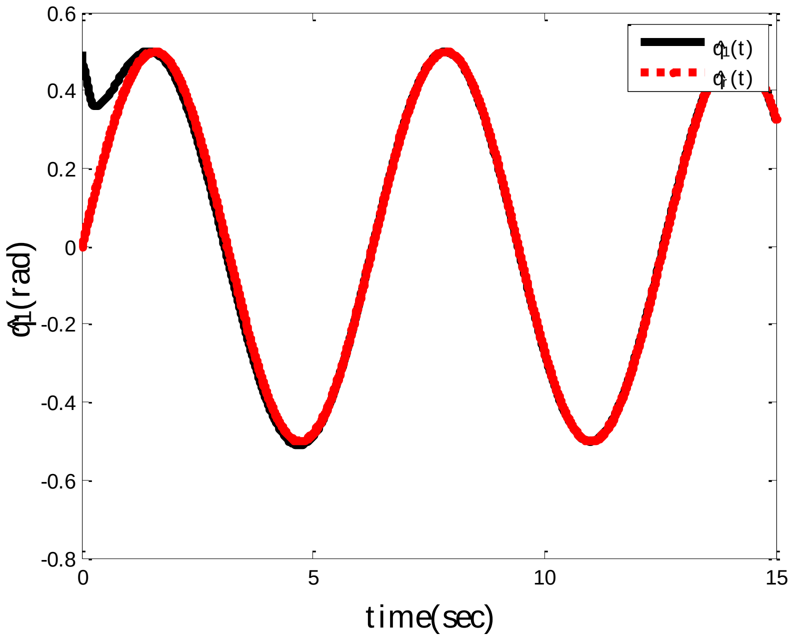


Figure 5-54 Time response of the desired trajectory to the estimated angular position 1 of the robot manipulator for case 2.

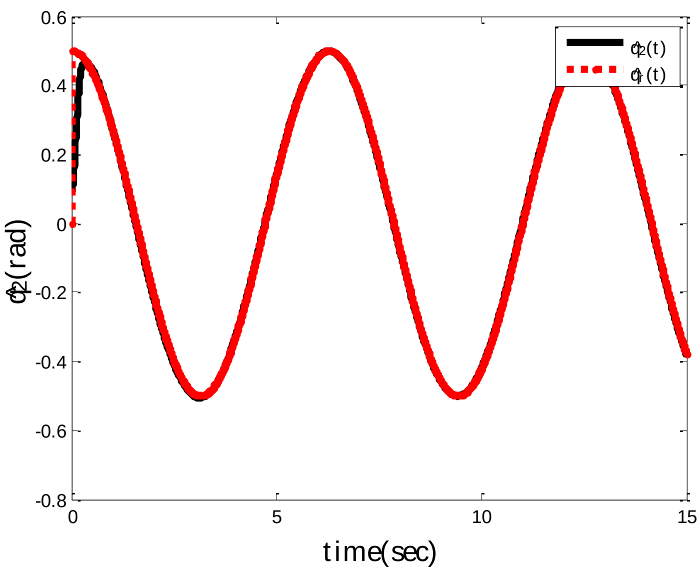


Figure 5-55 Time response of the desired trajectory to the estimated angular position 2 of the robot manipulator for case 2.

The graph shows the angular position of the robot manipulator $\hat{q}_1(t), \hat{q}_2(t)$ plotted with respect to desired trajectory. Here we can see that the robot manipulator is able to track the desired signal. The waveforms for estimated angular position are starting from 0.5 because of the implementation of the initial inside the system.

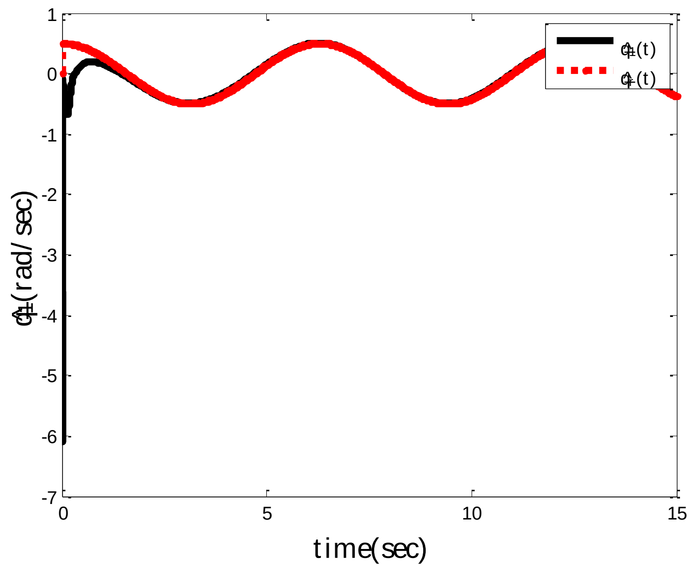


Figure 5-56 Time response of the desired trajectory and the derivative of estimated angular velocity 1 of the robot manipulator for case 2.

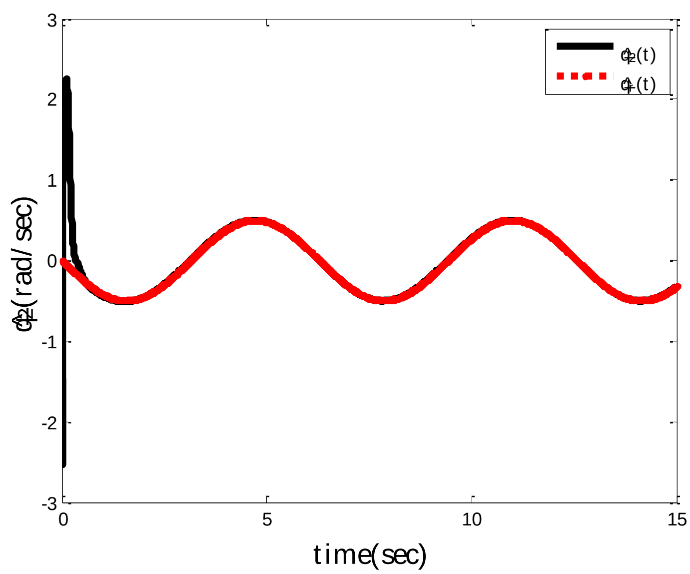


Figure 5-57 Time response of the desired trajectory and the derivative of estimated angular velocity 2 of the robot manipulator for case 2.

Because of the change in the initial conditions, we can see the variations at the start in the above waveforms corresponding to the angular velocity $\dot{\hat{q}}_1(t)$, $\dot{\hat{q}}_2(t)$ of the robot manipulator. Also, it is clear that with the change in initial conditions the manipulator is still able to track the desired trajectory.

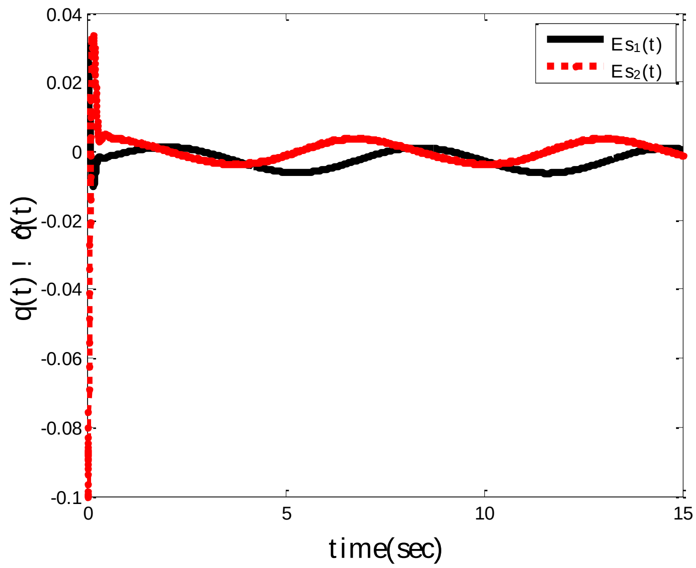


Figure 5-58 Convergence of position tracking error of the robot manipulator for case 2.

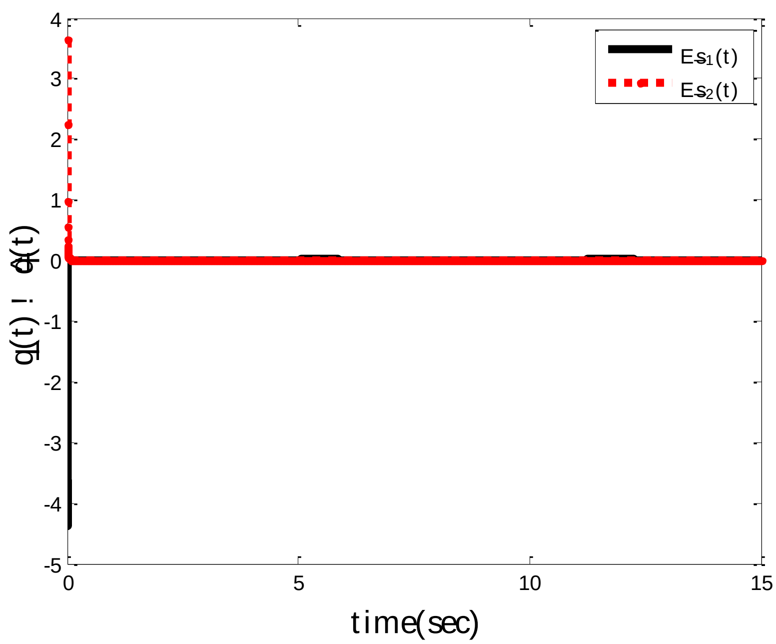


Figure 5-59 Convergence of velocity tracking error of the robot manipulator for case 2.

Form the above graphs it is clear that the convergence of position $q - \hat{q}$ and velocity $\dot{q} - \dot{\hat{q}}$ tracking error is achieved, the error as seen from the graphs are small and close to zero.

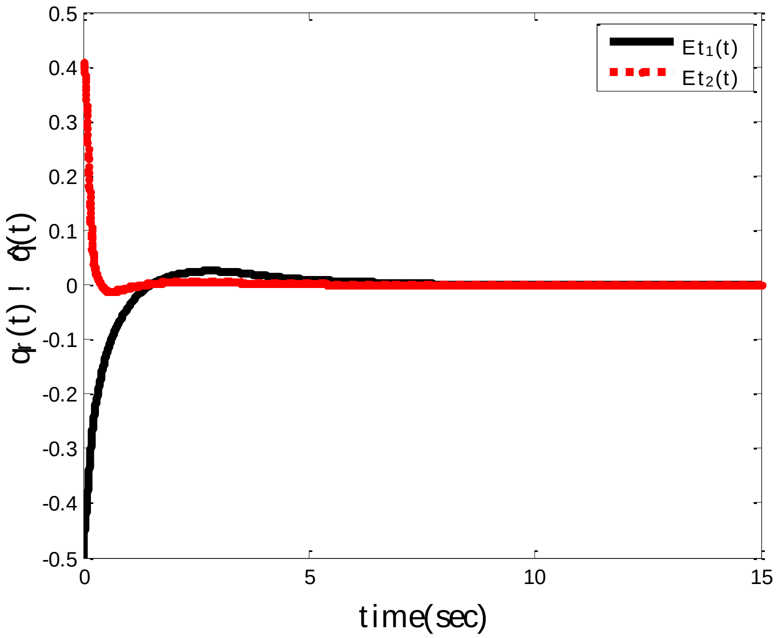


Figure 5-60 Robust Convergence of position tracking error of the robot manipulator for case 2.

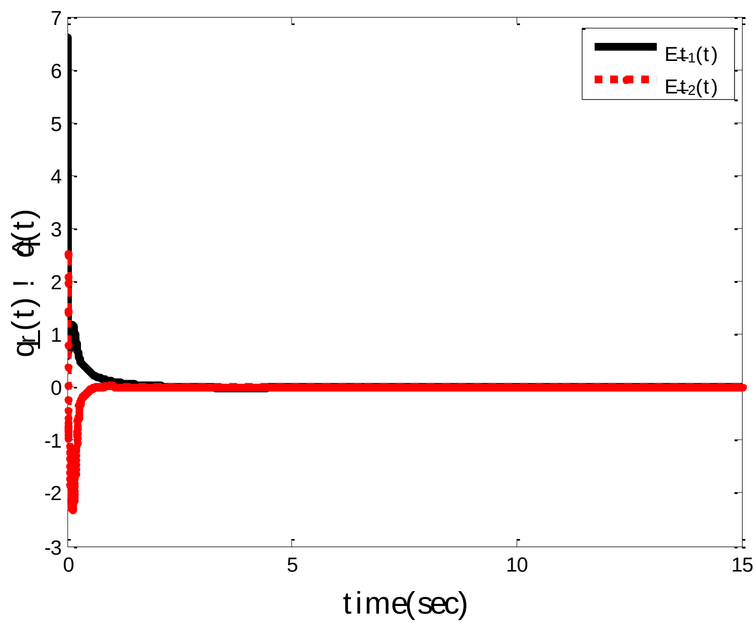


Figure 5-61 Robust Convergence of velocity tracking error of the robot manipulator for case 2.

Here graphs are plotted between estimated angular position and estimated angular velocity with respect to desired trajectory. The error difference them after a short duration dies down to zero, concluding the robust convergence property of the manipulator.

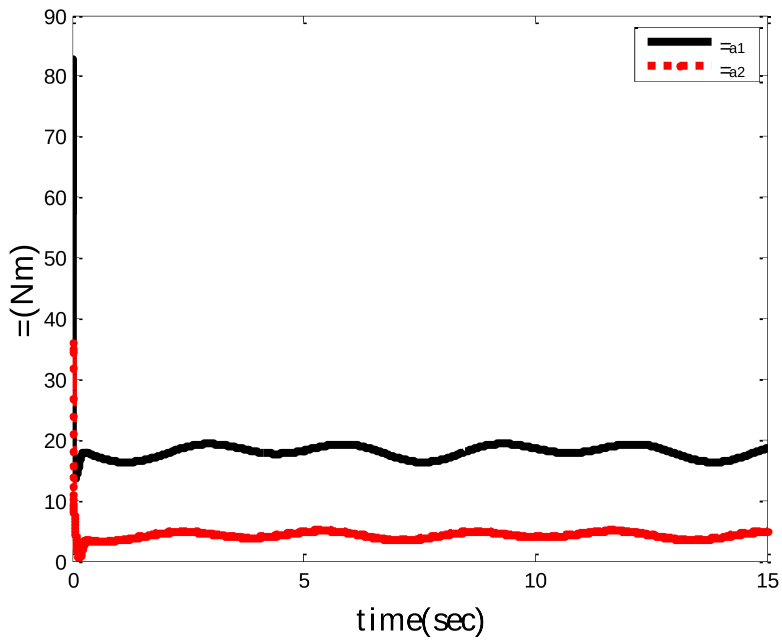


Figure 5-62 Applied input torques for case 2.

With the addition of the initial condition, there is no change the applied input torque of the manipulator, the wave is smooth without any chattering effect. Also, we can see that the control inputs $\tau_{a1}(t)(-)$ and $\tau_{a2}(t)(\cdots)$ are bounded and slightly oscillating within that bound.

Case 3: The performance of the closed loop system is evaluated here by applying external force without any initial condition.

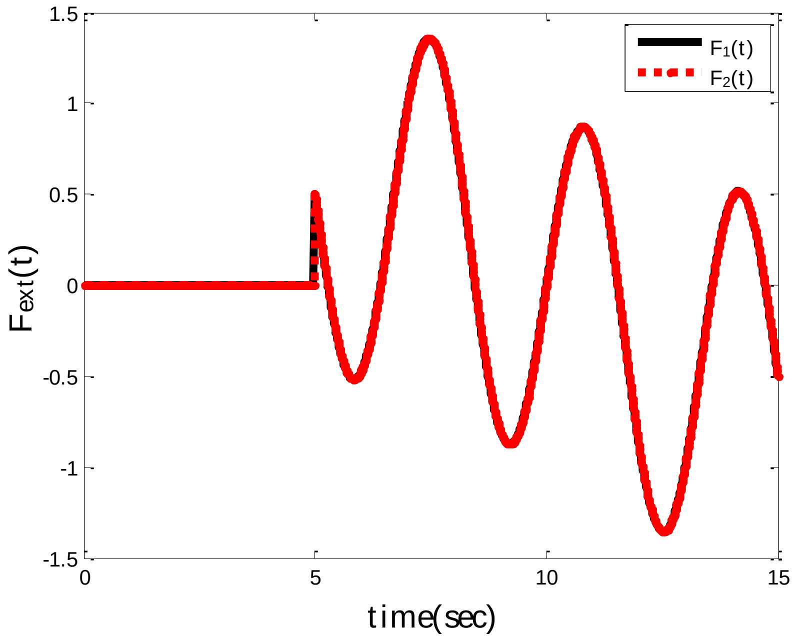


Figure 5-63 Time response of external force disturbances for case 3.

In this case we are adding external disturbance to the system and this disturbance is added at t=5 sec, this can be clearly seen from the above figure.

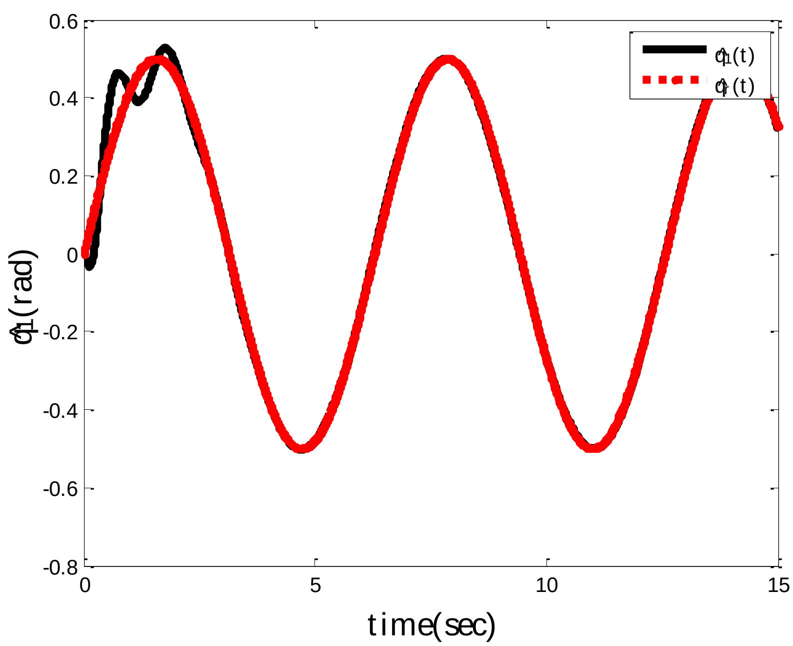


Figure 5-64 Time response of the desired trajectory to the estimated angular position 1 of the robot manipulator for case 3.

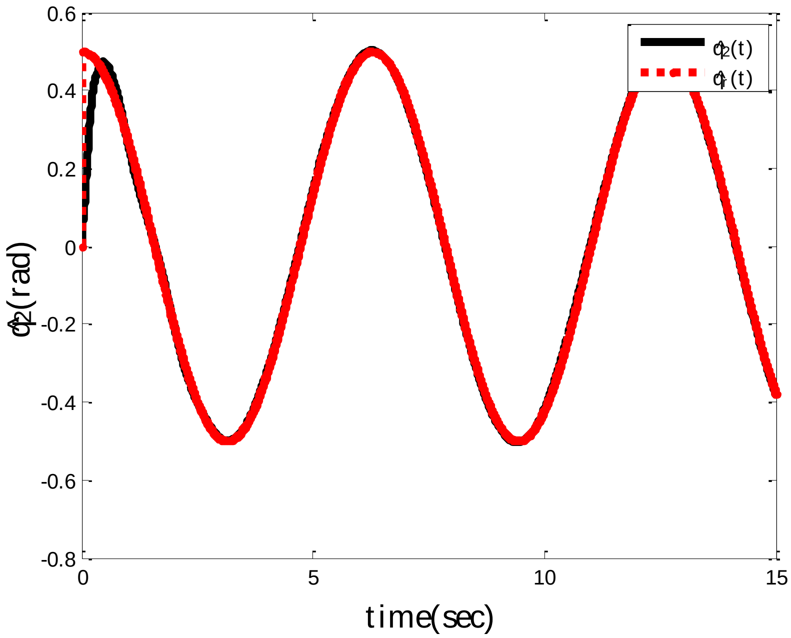


Figure 5-65 Time response of the desired trajectory to the estimated angular position 2 of the robot manipulator for case 3.

This graph explains the concept of trajectory tracking of the robot manipulator efficiently, in the above figures we can see that the time response of the desired trajectory to the estimated angular position of the robot manipulator is a perfect tracking phenomenon.

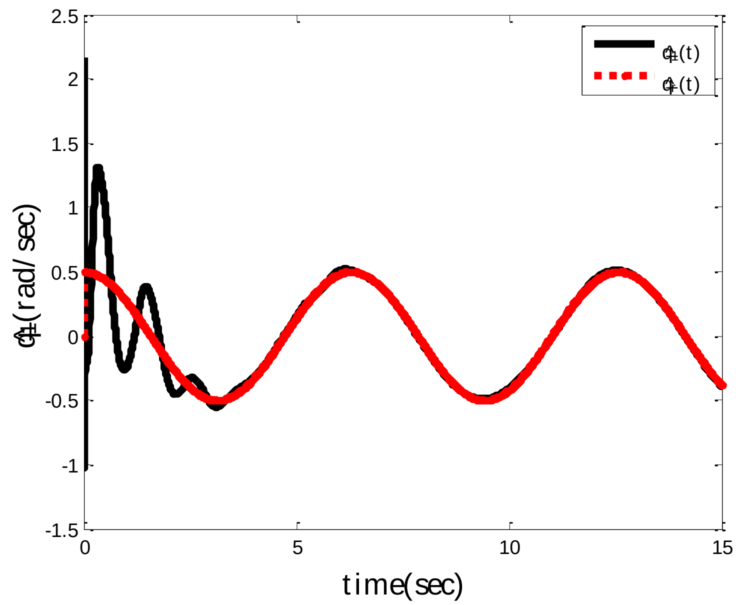


Figure 5-66 Time response of the desired trajectory and the derivative of estimated angular velocity 1 of the robot manipulator for case 3.

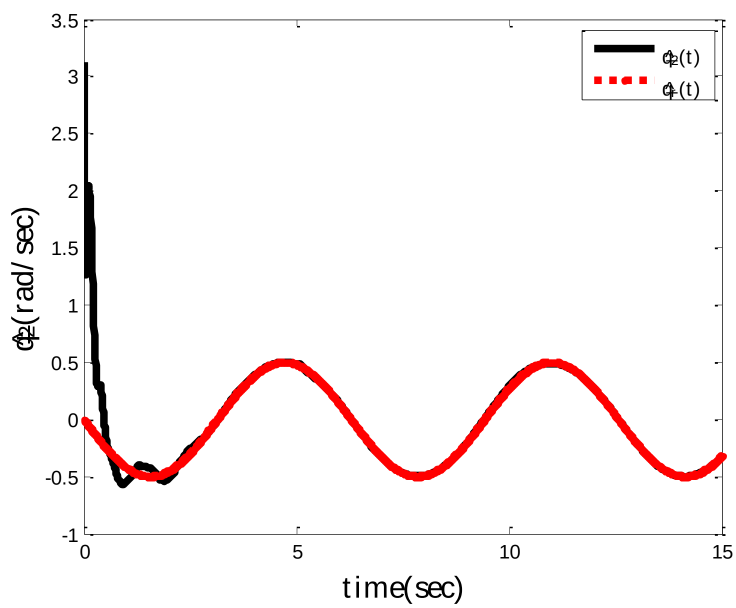


Figure 5-67 Time response of the desired trajectory and the derivative of estimated angular velocity 2 of the robot manipulator for case 3.

Time response of the desired trajectory $\dot{q}_r(t)$ with respect to the estimated response $\dot{\hat{q}}_1(t)$, $\dot{\hat{q}}_2(t)$ which is the angular velocity is plotted here and the robot manipulator is able to exhibit efficient tracking performance. The oscillations at the start is the time required by the manipulator to track the desired signal.

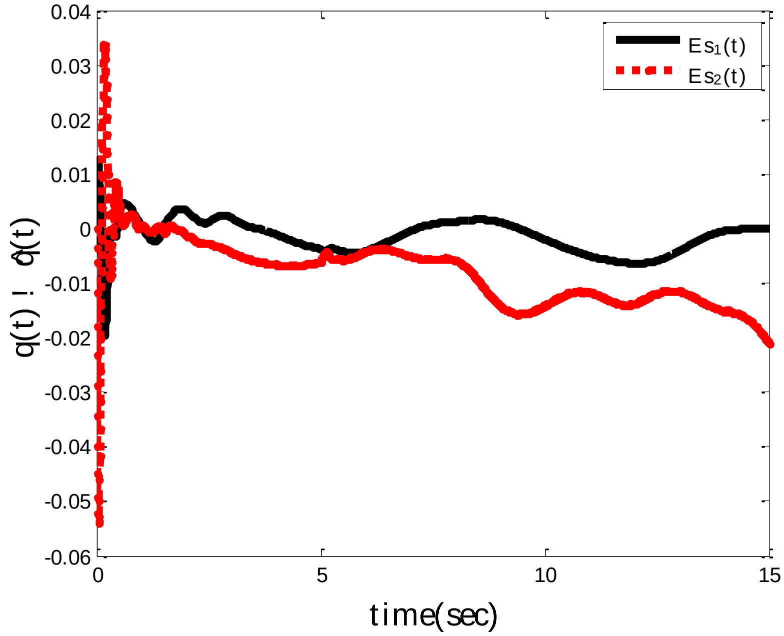


Figure 5-68 Convergence of position tracking error of the robot manipulator for case 3.

This graph explains the convergence of position tracking error phenomenon of the robot manipulator. Position tracking error is determined by taking the difference between the actual position of the manipulator to the estimated position of the manipulator. Here we can see that the tracking error is small and bounded close to zero.

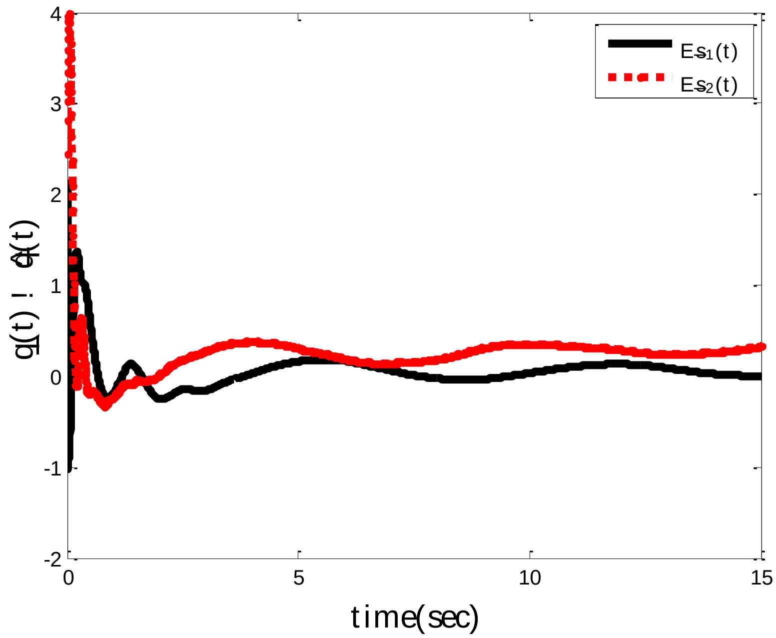


Figure 5-69 Convergence of velocity tracking error of the robot manipulator for case 3.

Here we can see that the velocity tracking error is very small and bounded close to zero.

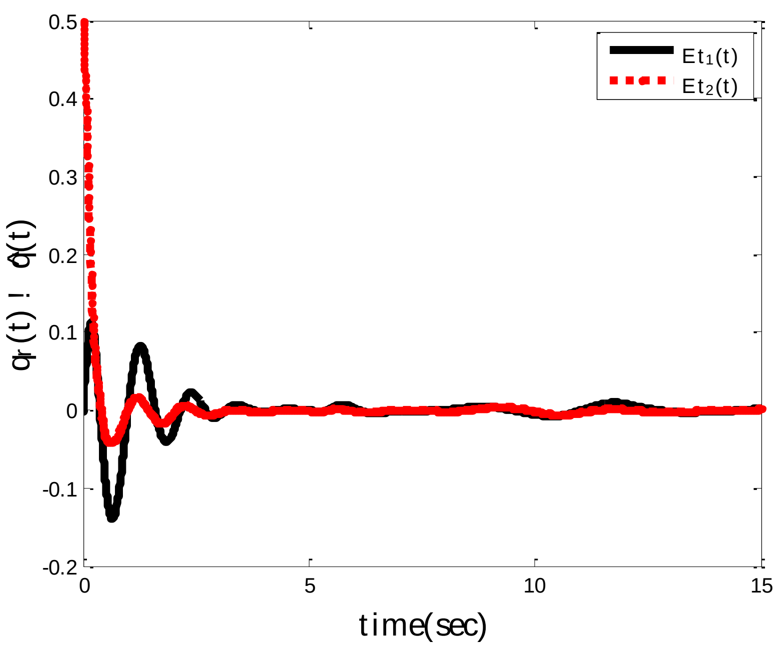


Figure 5-70 Robust Convergence of position tracking error of the robot manipulator for case 3.

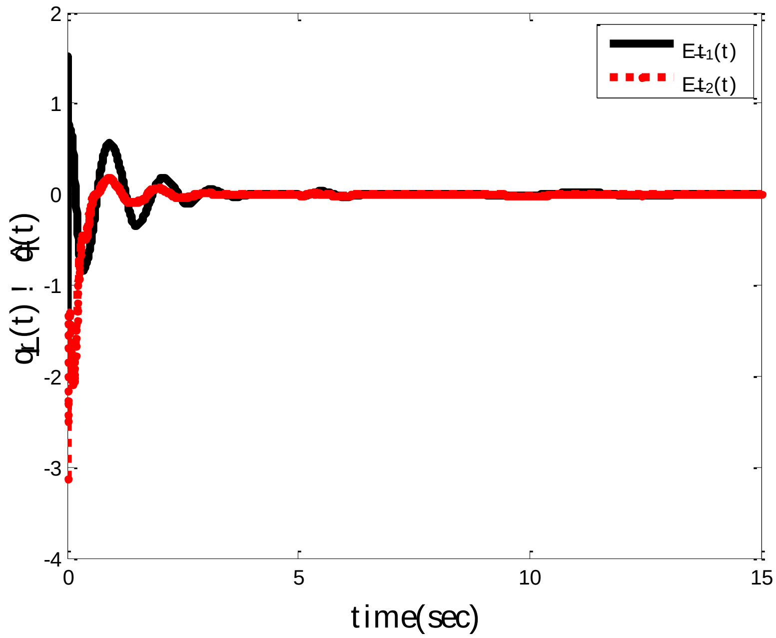


Figure 5-71 Robust Convergence of velocity tracking error of the robot manipulator for case 3.

To explain the concept of robust convergence phenomenon of the robot manipulator the above graphs are plotted. The first figure above is the difference between the desired position to the estimated position of the robot manipulator and it clearly shows that the error decays to zero, same is the case when we plot the graph between the desired velocity to the actual velocity of the manipulator, as this can be seen from the second figure shown above.

The control effort or the applied input torque required by the system is shown below, the graph can be seen with variations because of the implementation of external force and after a short duration they tend to oscillate within a control bound.

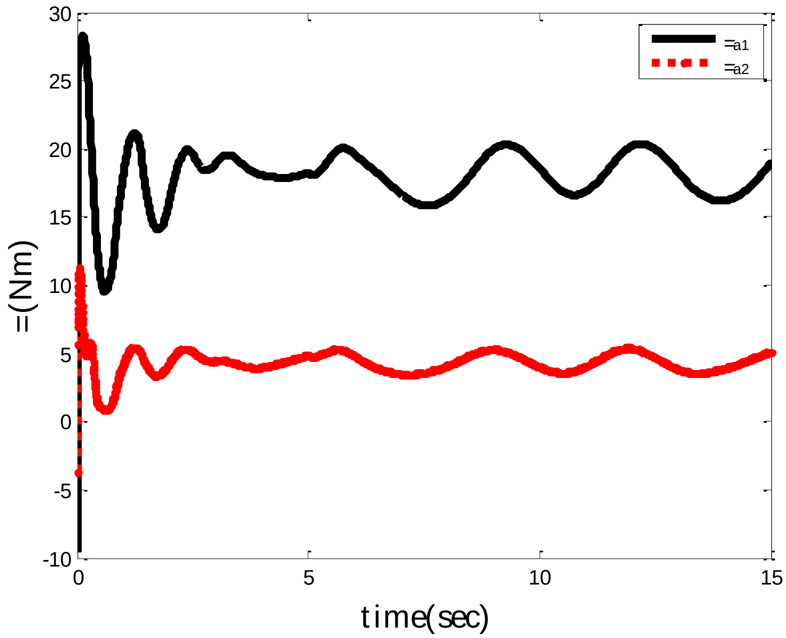


Figure 5-72 Applied input torques for case 3.

Case 4: The performance of the closed loop system is evaluated here by applying external force and initial condition.

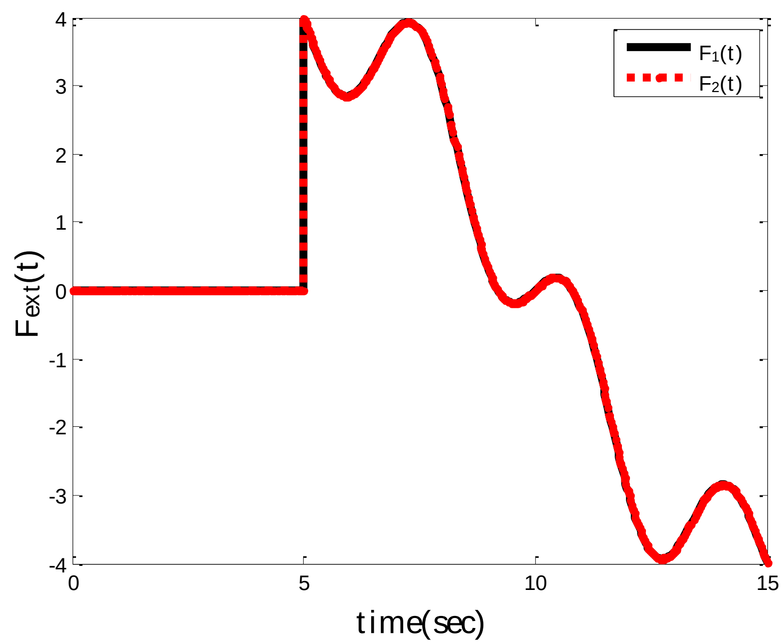


Figure 5-73 Time response of external force disturbances for case 4.

The applied external force disturbance can be seen above, the force is applied after t=5sec.

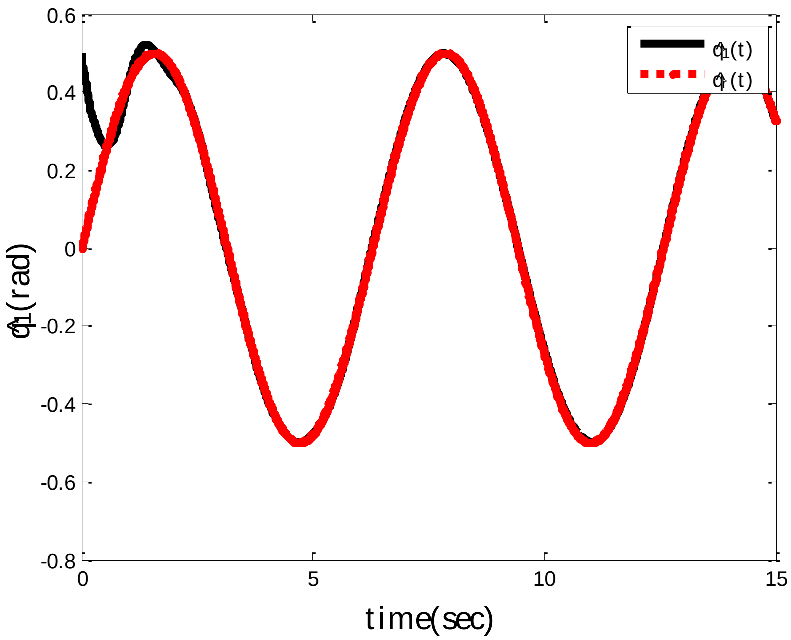


Figure 5-74 Time response of the desired trajectory to the estimated angular position 1 of the robot manipulator for case 4.

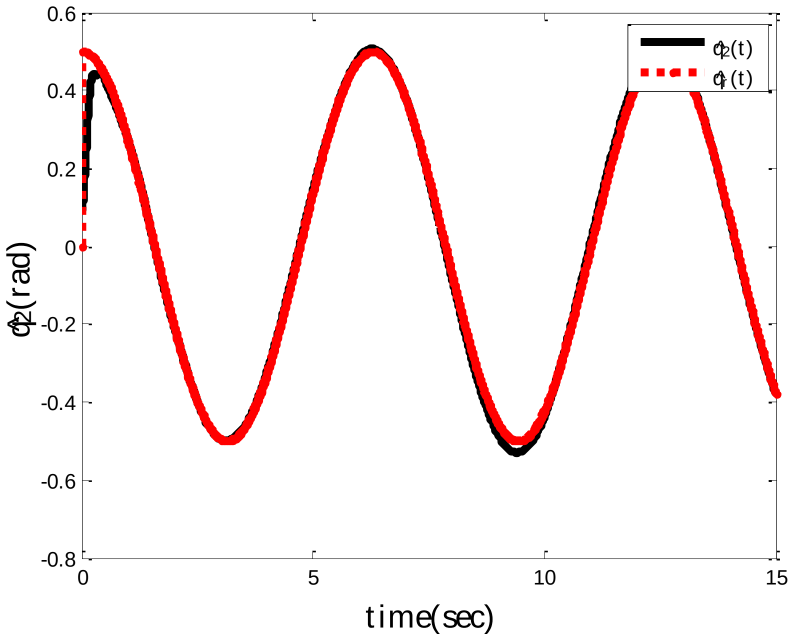


Figure 5-75 Time response of the desired trajectory to the estimated angular position 2 of the robot manipulator for case 4.

In order to validate the control law, these graph are plotted which shows the perfect tracking of the angular position $\hat{q}(t)$ of the robot manipulator with the desired behavior $q_r(t)$, slight variation at t=5 sec is due to implementation the disturbance signal. The waveforms have different starting point due change in the initial condition.

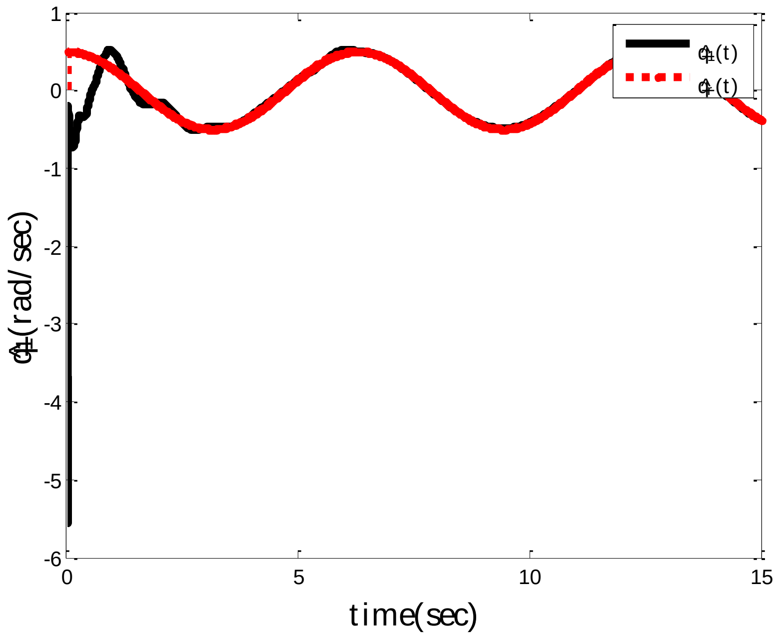


Figure 5-76 Time response of the desired trajectory and the derivative of estimated response angular velocity 1 of the robot manipulator for case 4.

The above figure corresponds to the angular velocity of the manipulator plotted with respect to the desired trajectory. Due to the external disturbance we see variations in the waveforms but within small duration the manipulator is again able to track the desired trajectory.

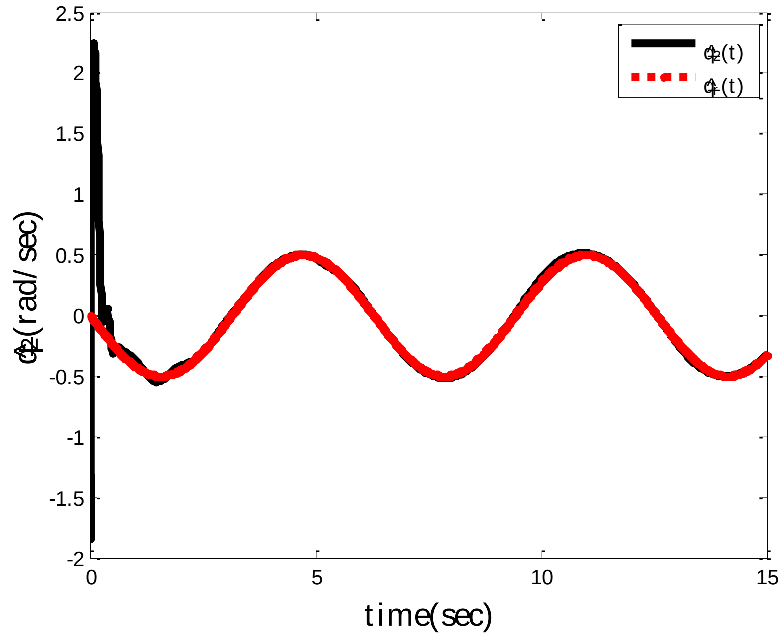


Figure 5-77 Time response of the desired trajectory and the derivative of estimated response angular velocity 2 of the robot manipulator for case 4.

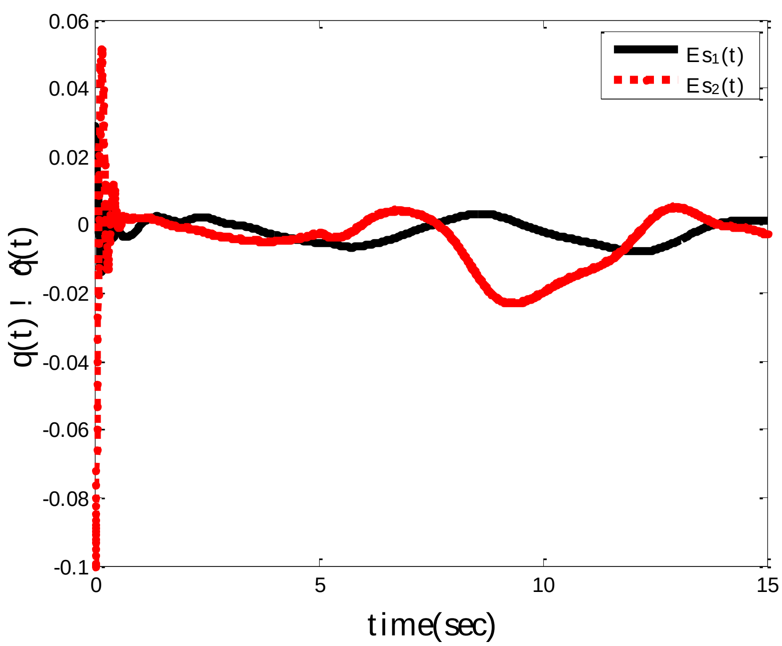


Figure 5-78 Convergence of position tracking error of the robot manipulator for case 4.

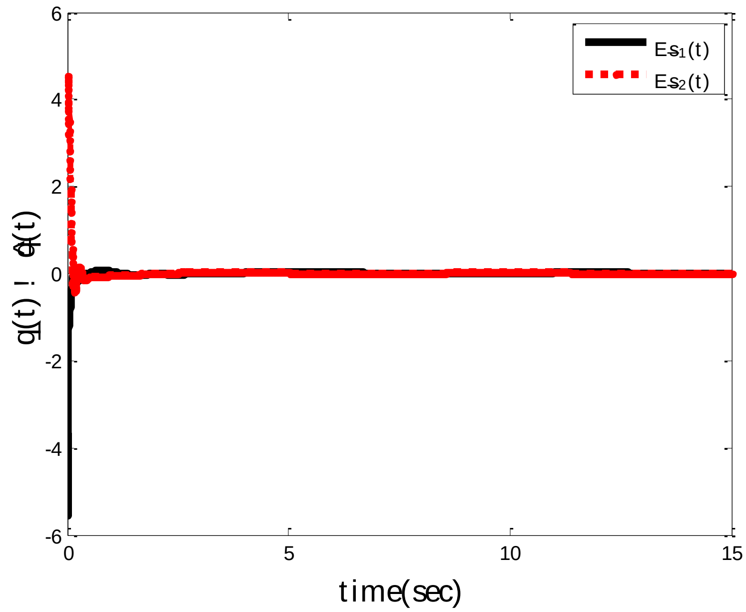


Figure 5-79 Convergence of velocity tracking error of the robot manipulator for case 4.

The difference between the actual position to the estimated position called as position tracking error is plotted showing convergence phenomenon. This result is same when angular velocity tracking error is plotted.

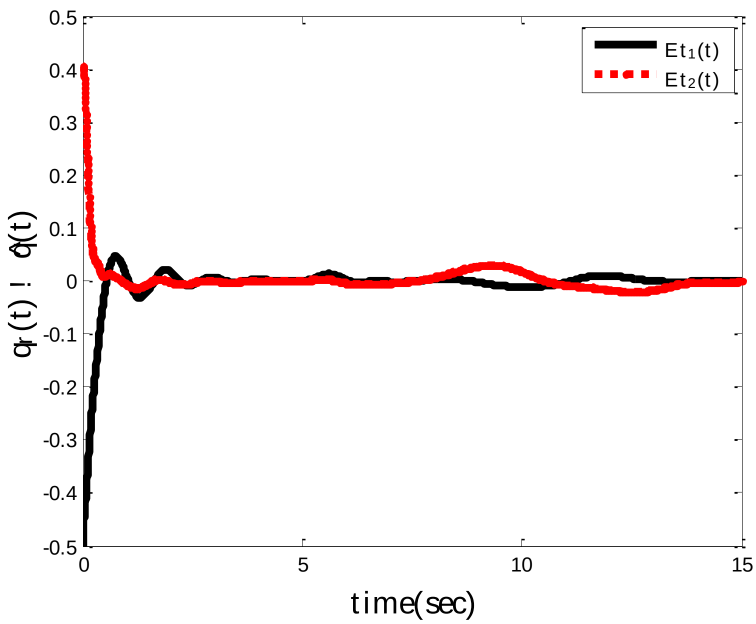


Figure 5-80 Robust Convergence of position tracking error of the robot manipulator for case 4.

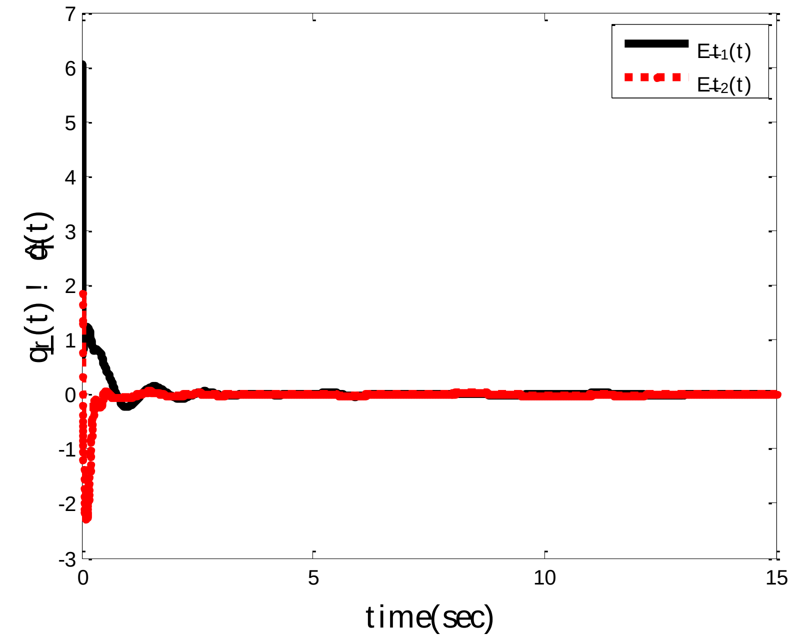


Figure 5-81 Robust Convergence of velocity tracking error of the robot manipulator for case 4.

The robust convergence of position and velocity tracking error of the robot manipulator is shown in the above figure.

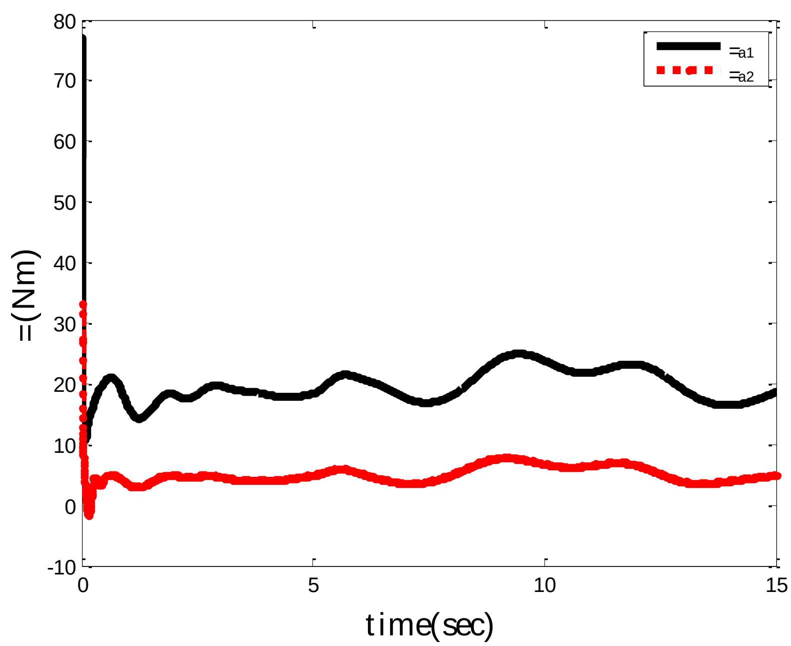


Figure 5-82 Applied input torques for case 4.

The required control effort also known as applied input torque $\tau_{a1}(t)(-)$ *and* $\tau_{a2}(t)(\cdots)$ shows a smooth behavior.

5.7 Comparing the Constant and Time varying High Gain Observer Based Robust Adaptive Control Scheme

In order to compare the results, we take a general case from both the control techniques for example, here we will analyze the results when both the robot manipulators are subjected to same initial condition and external forces, but the first manipulator will be controlled by a Constant High Gain Observer (CHGO) based control algorithm and the second manipulator will be controlled by Time Varying High Gain Observer (TVHGO) based control algorithm.

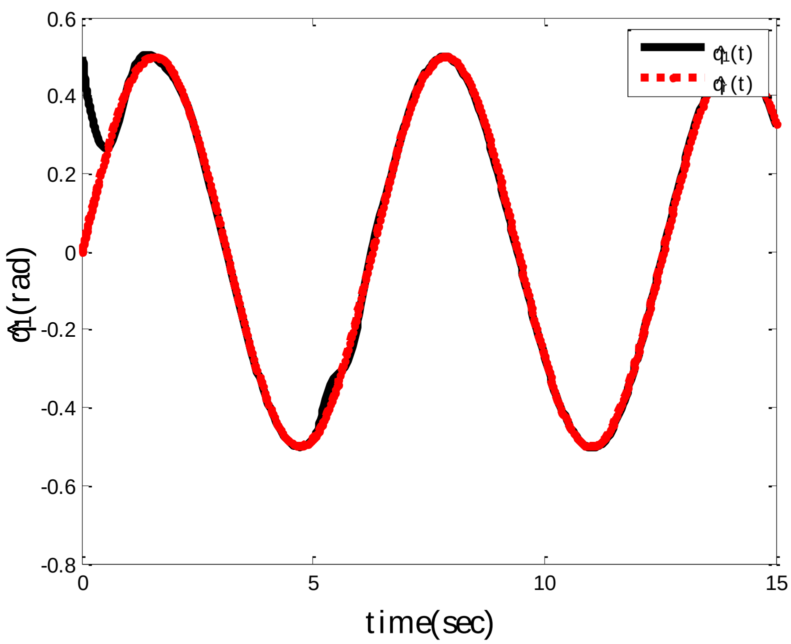


Figure 5-83 Time response of the desired trajectory to the estimated angular position 1 of the robot manipulator for CHGO.

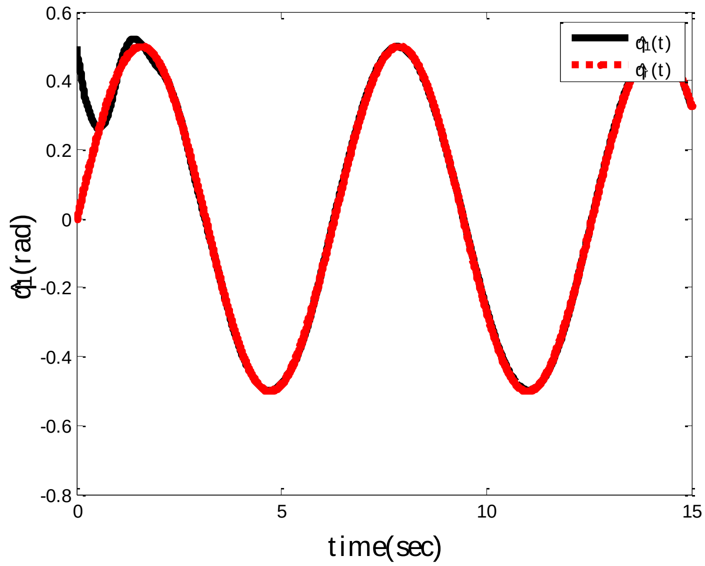


Figure 5-84 Time response of the desired trajectory to the estimated angular position 1 of the robot manipulator for TVHGO.

Time response of the desired trajectory $q_r(t)$ to the estimated angular position $\hat{q}_1(t)$ of the robot manipulator for both constant and time varying control law are plotted and it can be seen that the effect due to the disturbance on the constant high gain observer is more prominent.

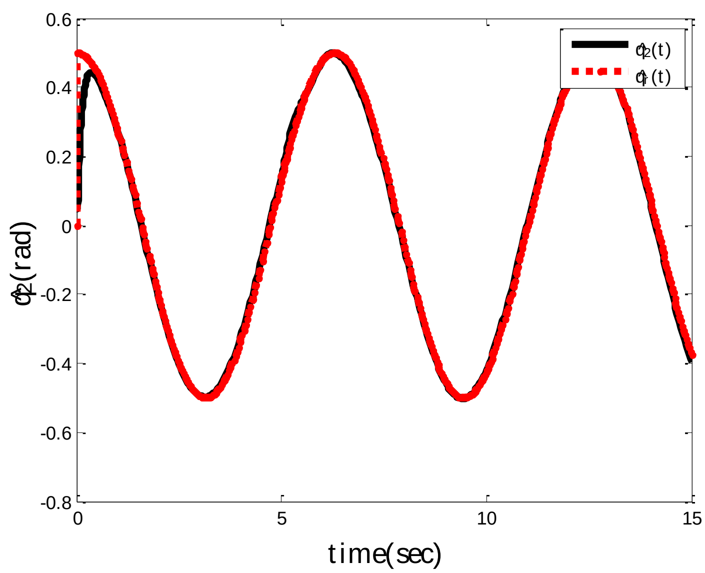


Figure 5-85 Time response of the desired trajectory to the estimated response angular position 2 of the robot manipulator for CHGO.

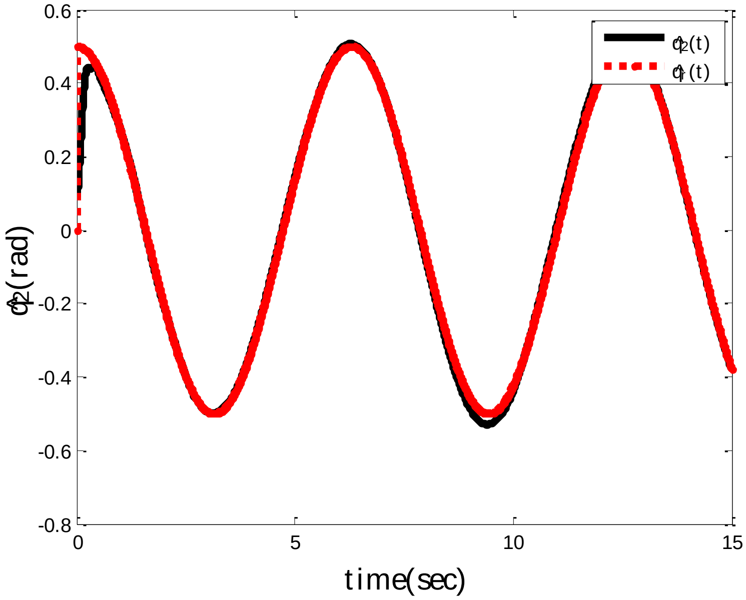


Figure 5-86 Time response of the desired trajectory to the estimated response angular position 2 of the robot manipulator for TVHGO.

The waveforms pertaining to the desired trajectory to the estimated angular velocity of the robot manipulator in both the cases are almost the same, moreover the waveform shows perfect tracking ability of the manipulator in both the cases.

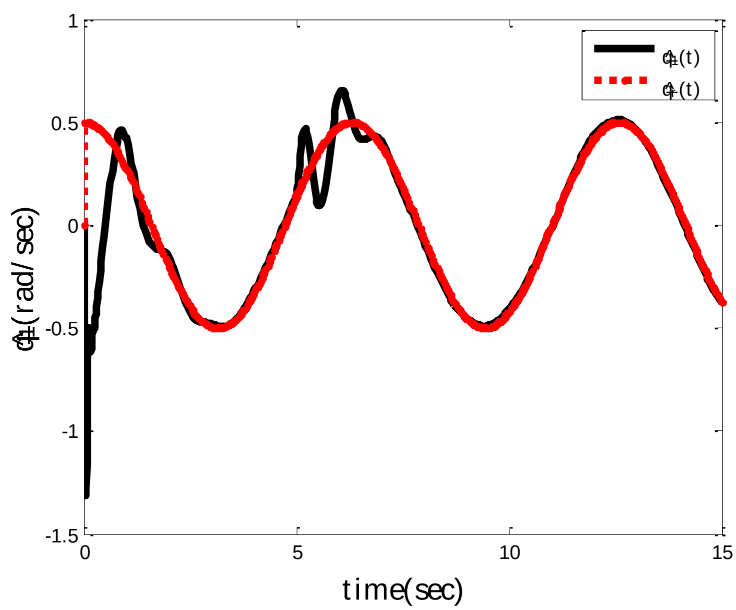


Figure 5-87 Time response of the desired trajectory and the derivative of angular velocity 1 of the robot manipulator for CHGO.

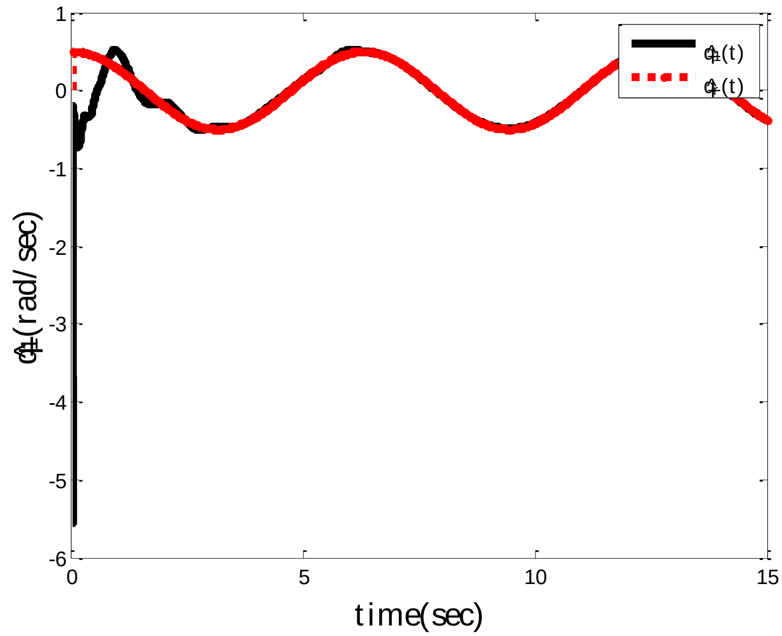


Figure 5-88 Time response of the desired trajectory and the derivative of angular velocity 1 of the robot manipulator for TVHGO.

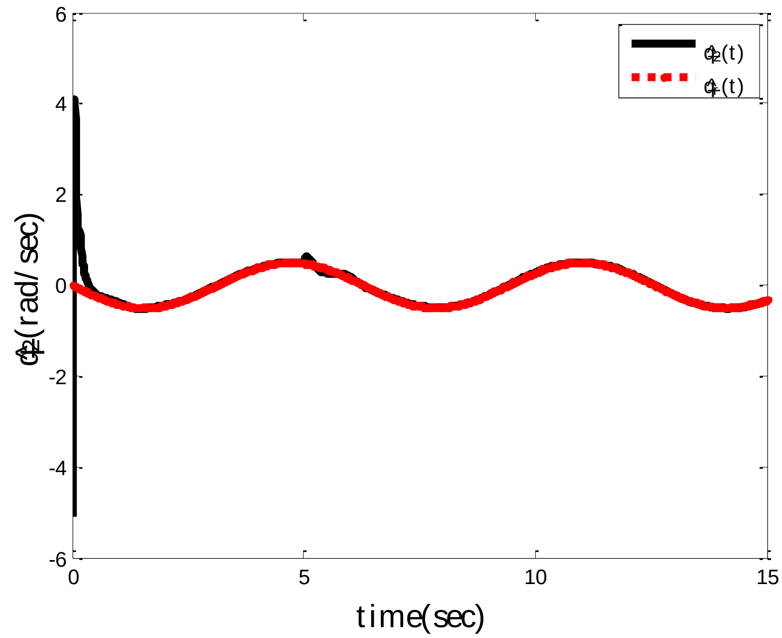


Figure 5-89 Time response of the desired trajectory and the derivative of angular velocity 2 of the robot manipulator for CHGO.

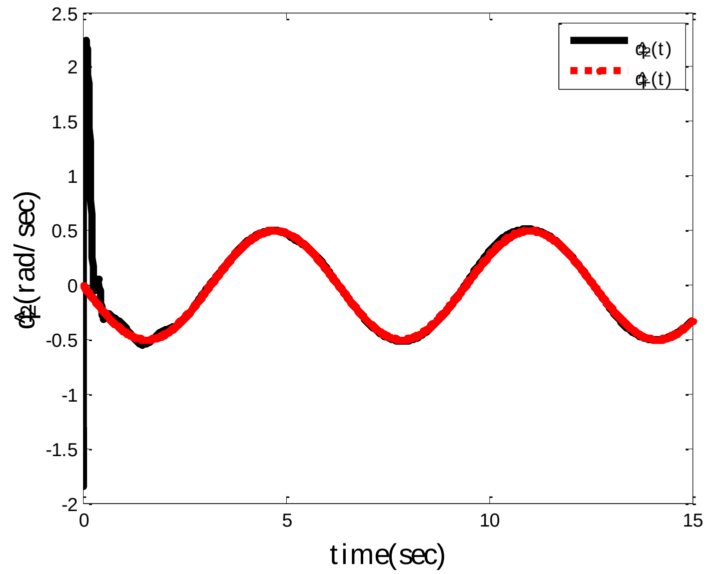


Figure 5-90 Time response of the desired trajectory and the derivative of angular velocity 2 of the robot manipulator for TVHGO.

The time response of the desired trajectory $\dot{q}_r(t)$ and the derivative of the estimated response $\dot{\hat{q}}_1(t)$ (angular velocity) of the robot manipulator using time varying high gain observer based control law is smooth with less oscillations when compared to the constant high gain observer based control law.

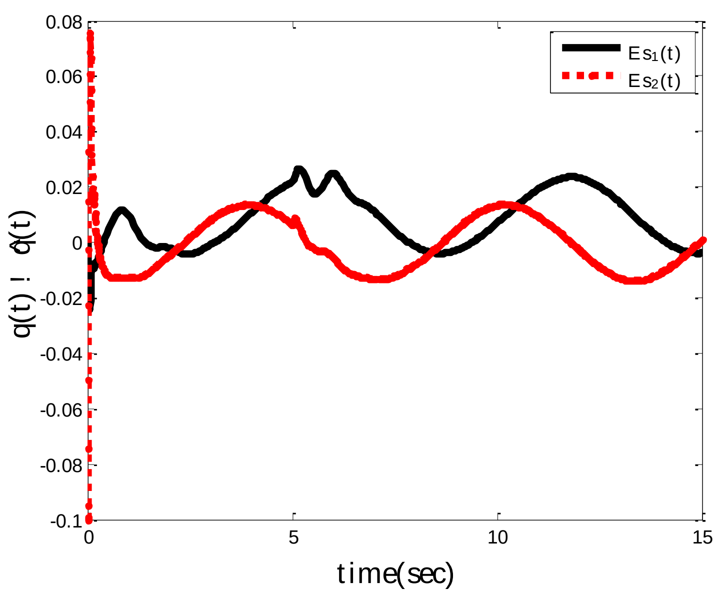


Figure 5-91 Convergence of position tracking error of the robot manipulator for CHGO.

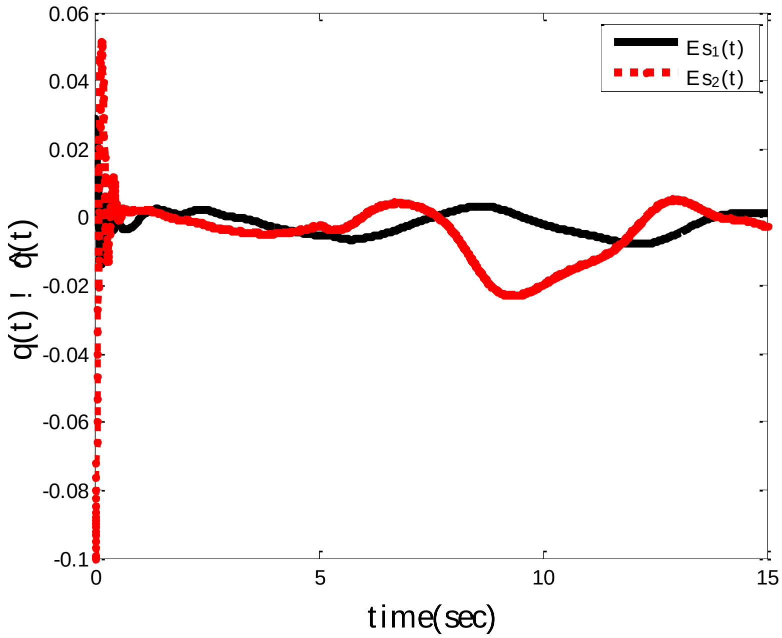


Figure 5-92 Convergence of position tracking error of the robot manipulator for TVHGO.

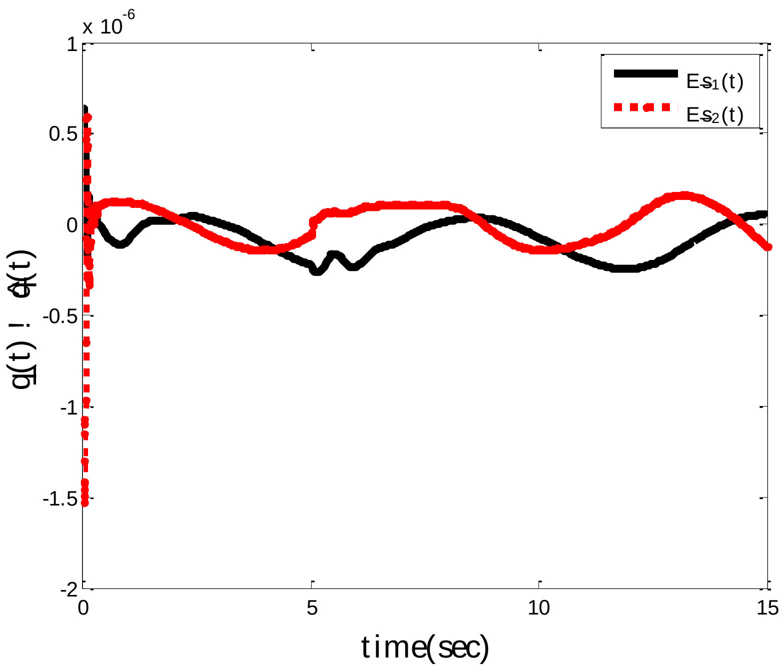


Figure 5-93 Convergence of velocity tracking error of the robot manipulator for CHGO.

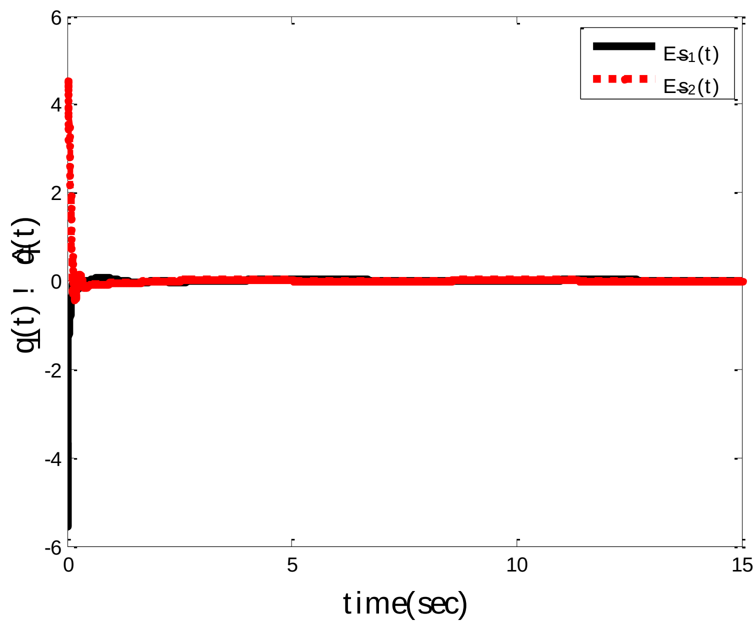


Figure 5-94 Convergence of velocity tracking error of the robot manipulator for TVHGO.

The convergence of velocity tracking error in case of constant high gain observer based control is of the order 10^{-6} which is negligible but in the time varying design the error is high but ensuring convergence property.

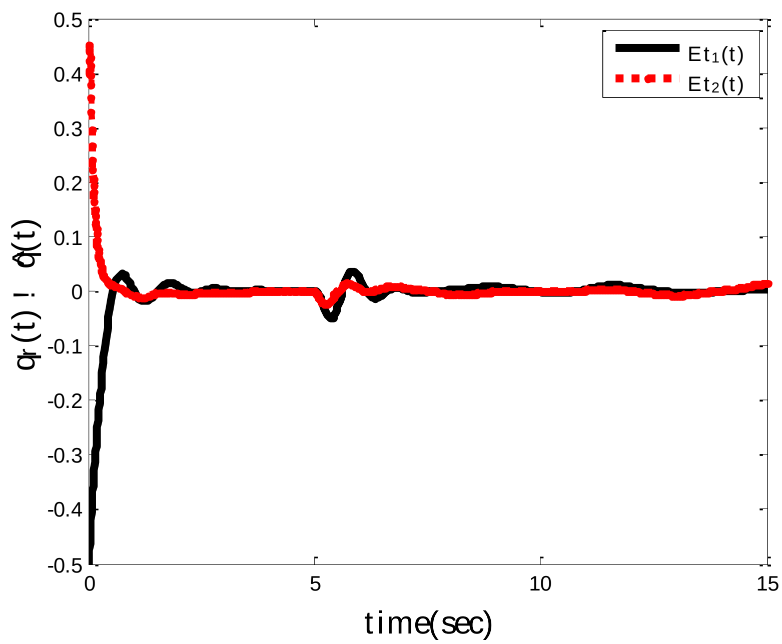


Figure 5-95 Robust Convergence of position tracking error of the robot manipulator for CHGO.

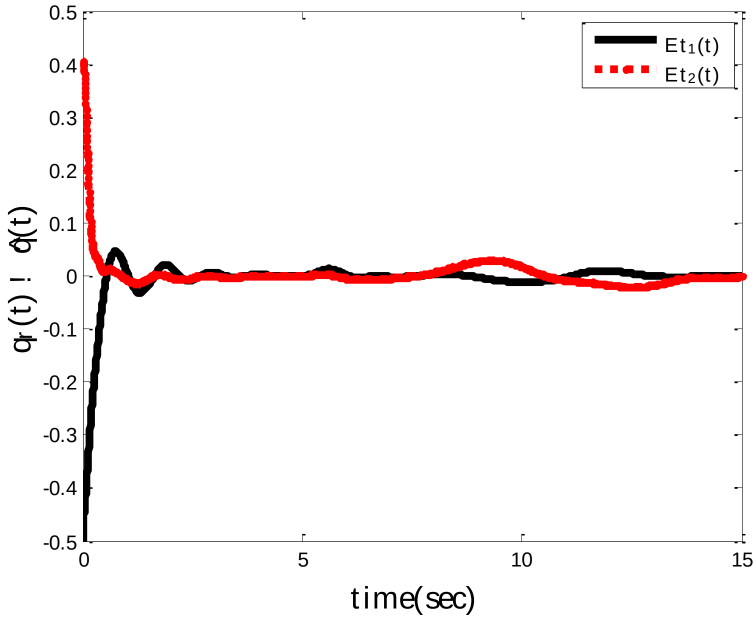


Figure 5-96 Robust Convergence of position tracking error of the robot manipulator for TVHGO.

The waveforms pertaining to the robust convergence of position error of the manipulator which are determined by taking the difference between the desired response with the estimated response are almost same in both the cases.

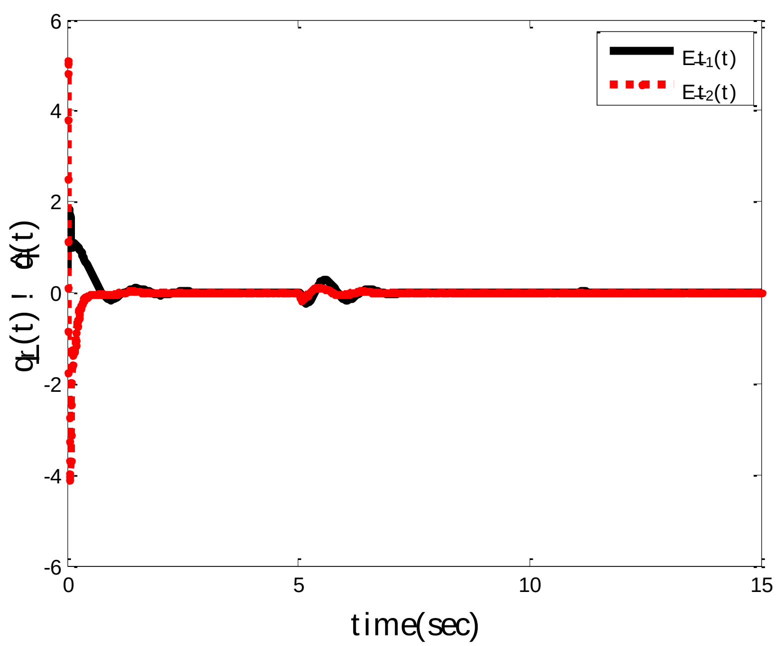


Figure 5-97 Robust Convergence of velocity tracking error of the robot manipulator for CHGO.

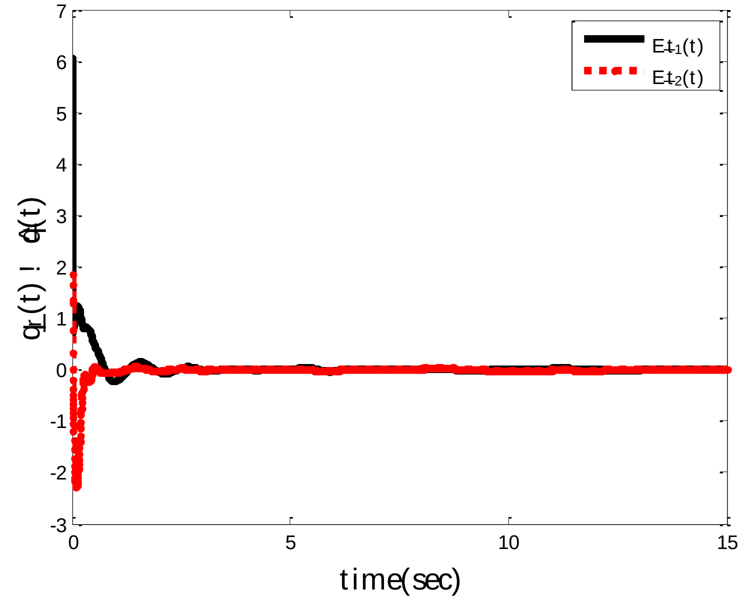


Figure 5-98 Robust Convergence of velocity tracking error of the robot manipulator for TVHGO.

The robust convergence phenomenon of velocity tracking error of the robot manipulator, more or less remains the same in both the cases, but in case of constant high gain observer design due to the effect of disturbance at t=5 sec, the effect can be seen clearly in the waveform.

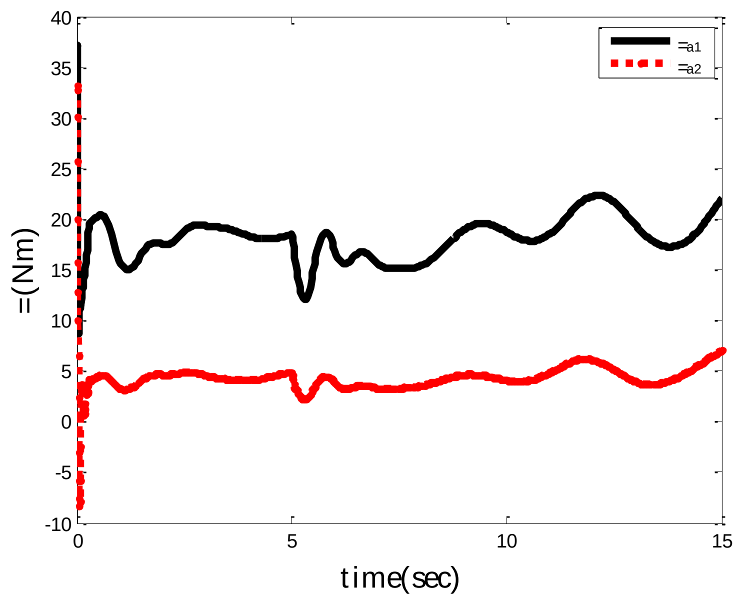


Figure 5-99 Applied input torques for CHGO.

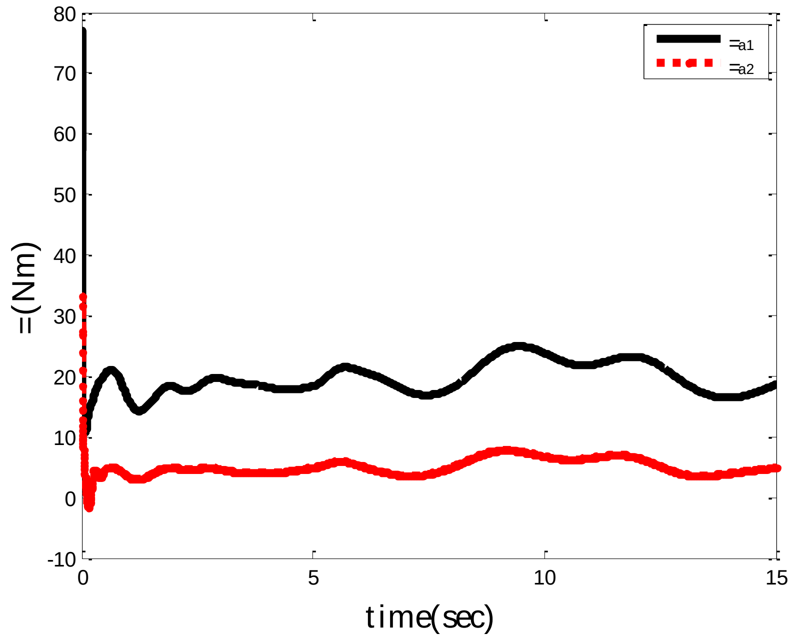


Figure 5-100 Applied input torques for TVHGO.

The control effort required in case of time varying observer based control is relatively higher when compared with the constant observer based control which can be clearly seen from the above figure. Also, the response of the time varying observer based control is smooth without much chattering, therefore exhibiting smoother control.

CHAPTER 6

SYNCHRONIZATION OF MULTIPLE ROBOT MANIPULATORS

6.1 Introduction

"An adjustment of rhythms of oscillating objects due to their weak interactions"

– Christian Huygens

Robotics research in recent years focuses on multi-robot systems, since a single robot is no longer the best solution for applications domains such as robotic exploration in hazardous environments, automated production plants, autonomous robot colonies for operations in remote locations and agricultural purposes. The use of multi-robots becomes a popular alternative to single robot for a variety of difficult robotic tasks, such as planetary exploration or flexible automation, since they are both fault tolerant (due to redundancy) and faster (due to parallelism) than single robots [164].

Also there are other factors that make them more prominent such as:

(i) Robot team can accomplish a given task more quickly,

(ii) Robot team could make effective use of skills,

(iii) Team of robots can localize themselves more efficiently,

(iv) A Team of robots generally provides a more robust solution,

(v) A Team of robots can produce a wider variety of solutions.

Furthermore researchers argue that increasing the number of robots without proper justification would not result in efficient solution. The best solution for a team of robots can be achieved only by coordinating the multi-robot system in synchronous environment. Thus, the problem of synchronous coordination of multi-robot systems has risen to the forefront of robotics research in recent years [165]. To get high quality performance from synchronized multi-robot systems, the systems must include the following performances:

(i) Robustness,

(ii) Quickly adapting to change in dynamical conditions,

(iii) Ability to deal with limited and imperfect knowledge of the states of the dynamic system,

(iv) Ability to deal with the heterogeneous teams of robots.

Keeping in mind the above performance parameters, we concentrate on synchronization of multi-robot manipulators using robust adaptive control schemes along with observer to deal with the limited information of the states of the systems.

Synchronization is a word, coined from two Greek words chronous (meaning time) and sign (meaning common) to imply occurring at the same time. This effect in physical systems was first observed by Christian Huygens who found that two clocks supported from a common wooden support had the same rhythmic motion. Even when the clocks were disturbed they reestablished their rhythms. Later this phenomenon was found and investigated in different man made devices like electronic generators, musical instruments etc. Nature employs synchronization at different levels in biological systems. Synchronous variation of nuclei, synchronous firing of neurons, adjustment of heart rate with respiration, synchronous flashing of fire flies etc. are some such examples of natural phenomenon.

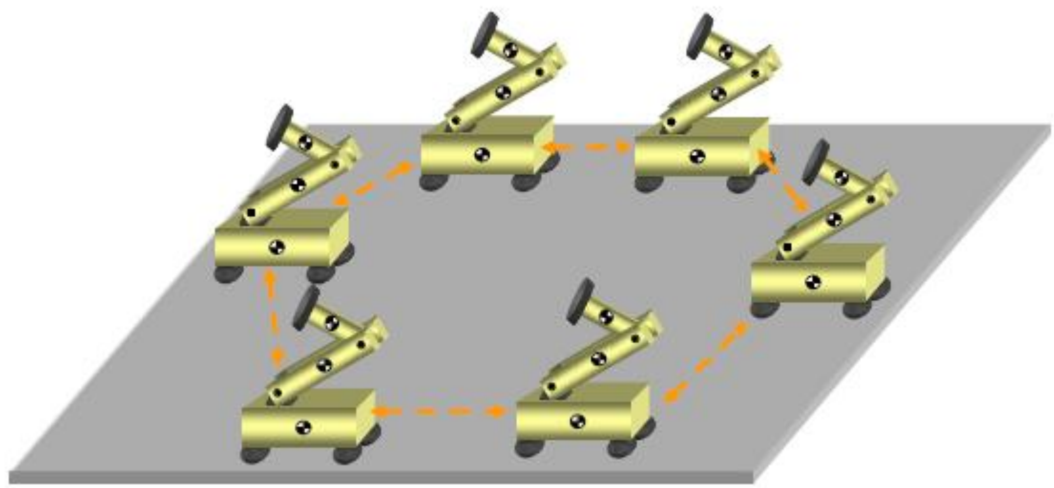


Figure 6-1 Synchronization of multi-robots systems [166].

6.2 Types of Synchronization

There are a variety of different types of synchronization that can occur depending on the nature of the system and the nature of the coupling:

(i) Total or complete synchronization,

(ii) Lag synchronization,

(iii) Phase synchronization and

(iv) Generalized synchronization.

The specific characteristics of each of these is discussed below:

6.2.1 Complete Synchronization

Consider two systems denoted X and Y. One way they can synchronize when coupled is when the state variables x and y become completely identical,

$$\lim_{t\to\infty}\|x-y\| \to 0 \tag{6.1}$$

This is referred to as a complete or identical synchronization and represents a special case of all types of synchronization.

6.2.2 Lag Synchronization

In this case the state variables x and y of the systems X and Y are identical but at different times, namely

$$x(t+\tau) = y(t) \tag{6.2}$$

6.2.3 Phase Synchronization

Phase synchronization is defined as the locking of the individual phases of the oscillators X and Y,

$$|n\phi_x(t) - m\phi_x(t)| = \text{Constant} \tag{6.3}$$

Where n and m are integers. Note that the amplitude can be different; if the amplitude are identical, this reduces to lag synchronization. This is the oldest concept of synchronization, which was originally introduced for the description of coupled harmonic oscillators [128]. This concept has been extended to nonlinear and even chaotic systems [166] and has been applied to many biological and other systems [128].

6.2.4 Generalized Synchronization

Consider uni-directionally coupled systems (skew products systems),

$$\dot{x} = f(x) \tag{6.4}$$

$$\dot{y} = g(y, x) \tag{6.5}$$

Generalized synchronization is said to occur when the states of the driven system is completely determined by the state of the driving system. Mathematically, this is given by the existence of the unique functional relationship,

$$y = H(x) \tag{6.6}$$

Where the function H can be smooth or non-smooth [126].

When systems are synchronous, the overall dynamics occurs in a subspace in the phase space termed the synchronization manifold, which is invariant under the flow. Orbits that start out in the synchronization manifold remain in it for all time. In systems having natural symmetry, the symmetric states form an invariant manifold. When coupling between the systems leads to phase space contraction, trajectories are attracted towards the manifold. As a result the Lyapunov exponent transverse to the manifold is negative, and this is a sufficient condition for synchronization to occur.

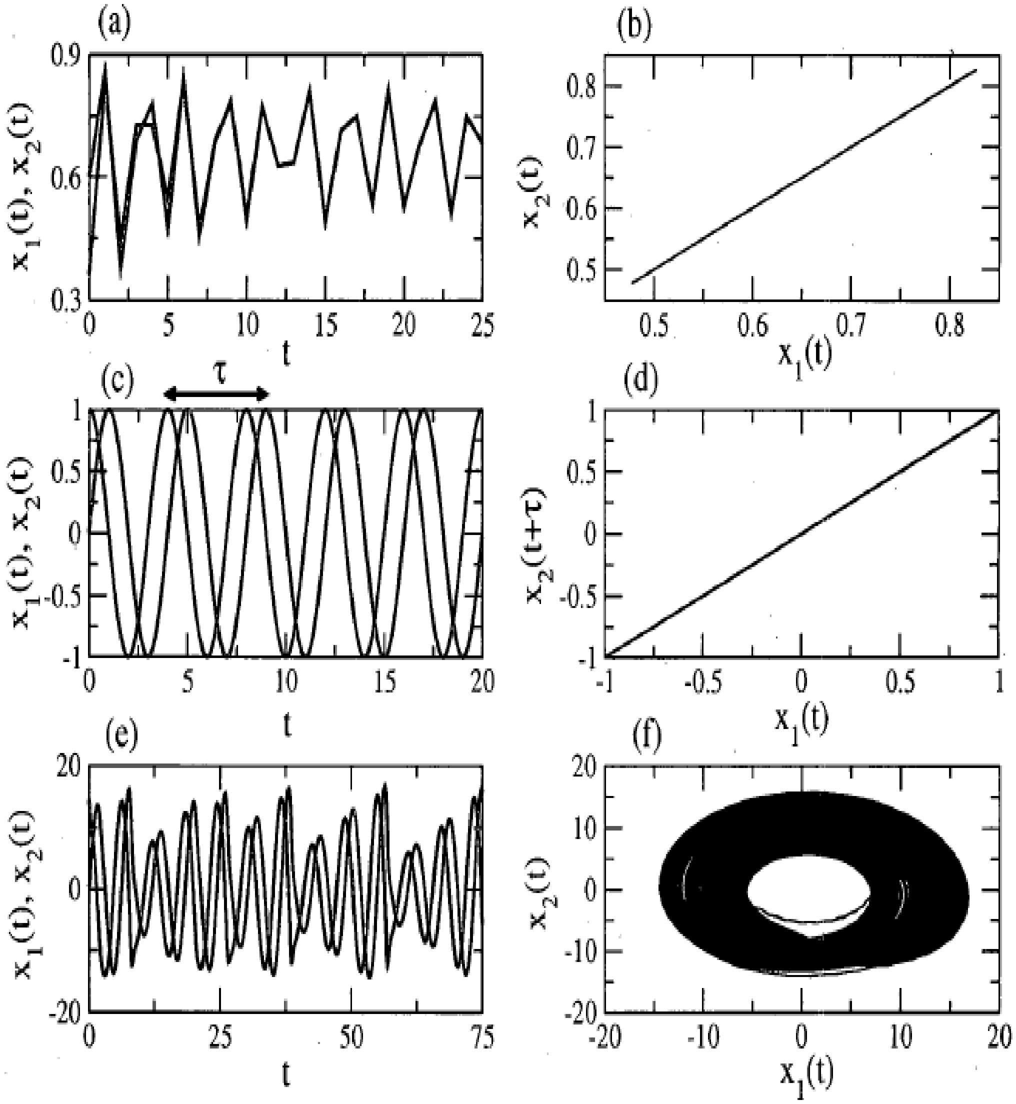


Figure 6-2 Pictorial description of different types of synchronization [126].

(a) and (b) represents complete synchronization,

(c) and (d) represents lag synchronization and

(e) and (f) represents phase synchronization.

(a), (c) and (e) show the respective time-series and

(b), (d) and (f) shows the respective synchronization manifolds.

6.3 The Synchronization Concept

Assume a multi-agent system with n agents given to perform a task that requires the coordination of each agent's motion, i.e., an n-DOF robotic manipulator following a defined contour. The goal of synchronization control is to regulate and synchronize the motion of all agents so that a certain kinematics relationship is maintained for the agents as is required by the coordination task. Not unlike a sliding surface, the regulation of the agents to maintain a kinematic relationship can be understood as guiding the agents along the boundary of multidimensional a compact set. A time-varying desired shape in such a compact set can be introduced as $S(\rho, t)$ with ρ being a state vector and the time. Then the boundary of the shape can be parameterized by a curve, denoted as $\partial S(\rho, t)$.

Therefore, the synchronization of a system of multiple entities requires two main tasks to be performed. The first one is the classic control goal of bringing the state error of each agent as close to zero as possible as time increases, i.e., $e_i \to 0$ as $t \to \infty$. The second task requires maintaining each agent on the desired curve so that $\partial S(\rho, t)$=0. Both tasks are equally important and should be achieved simultaneously in order to achieve system synchronization. Although the constraints for the second synchronization task can take various forms, depending on the system's nature and purpose, the following mathematical expression can be used without loss of generality:

$$\partial S(x_i, t) = 0: \{x_i(t) = A_i(t)C(t) + B_i(t)\}, i = 1,2, \ldots\ldots n \qquad (6.7)$$

Where $x_i(t)$ is the state of the ith agent, $A_i(t)$ is a constraint matrix based on the desired boundary, which on itself depends on the agents characteristics; $C(t)$ is a common vector of characteristics that are applicable on the whole system but not on individual agent, and $B_i(t)$is an offset of the ith agent. The above equation indicates that all agents of the system can be related through the common vector $C(t)$ which requires a linear mapping from $x_i(t)$ to $C(t)$ denote as $\{x_i(t)\} \rightarrow C(t)$. For the existence of this unique linear mapping to exist, the constraint matrix $A_i(t)$ must be invertible. It should be noted that because $A_i(t)$ is based on the desired boundary, it is mainly determined by the topology of the given formation task.

Assuming the existence of the inverse of matrix $A_i(t)$, it follows that:

$$C(t) = A_n^{-1}(t)(x_i(t) - B_i(t)) \tag{6.8}$$

However, $C(t)$ is a common vector for every agent. Hence for a system of n agents:

$$A_1^{-1}(t)\big(x_1(t) - B_1(t)\big) = \cdots\cdots = A_n^{-1}(t)\big(x_n(t) - B_n(t)\big) = C(t) \tag{6.9}$$

Equation (6.9) is expressed in terms of the actual state $x_i(t)$ of the ith agent, but it can also be expressed in terms of the desired state $x_i^d(t)$,

$$A_1^{-1}(t)\left(x_1^d(t) - B_1(t)\right) = \cdots\cdots = A_n^{-1}(t)\left(x_n^d(t) - B_n(t)\right) = C(t) \tag{6.10}$$

Subtracting Eq (6.9) from (6.10)

$$A_1^{-1}(t)e_1(t) = A_2^{-1}(t)e_2(t) = \cdots\cdots = A_n^{-1}(t)e_n(t) \tag{6.11}$$

Where e is the tracking error.

Lastly, $C_i(t) = A_i^{-1}(t)$ can be defined as the coupling parameters of the ith agent, leading to:

$$C_1(t)e_1(t) = C_2(t)e_2(t) = \cdots\cdots = C_n(t)e_n(t) \tag{6.12}$$

The above equation is ultimately the synchronization control goal which all position error must satisfy in order for system to meet the coordination requirements.

It should be noted that the coupling parameters do not always take the form of mathematical expressions, but they can simply be constants or identity matrices. Additionally, depending on the nature and purpose of the system, all the agents might have the same coupling parameter. In that case, the synchronization goal becomes:

$$e_1(t) = e_2(t) = \cdots\cdots = e_n(t) \tag{6.13}$$

Obviously, such is the case for robot manipulators which we are working on.

6.4 Synchronization Control of Robot Manipulators

Synchronization control of robot manipulators is beneficial for industry as it allows more than one manipulator to do complex tasks together. For example, assembling of products or lifting of heavy objects can be done with higher efficiency and precision.

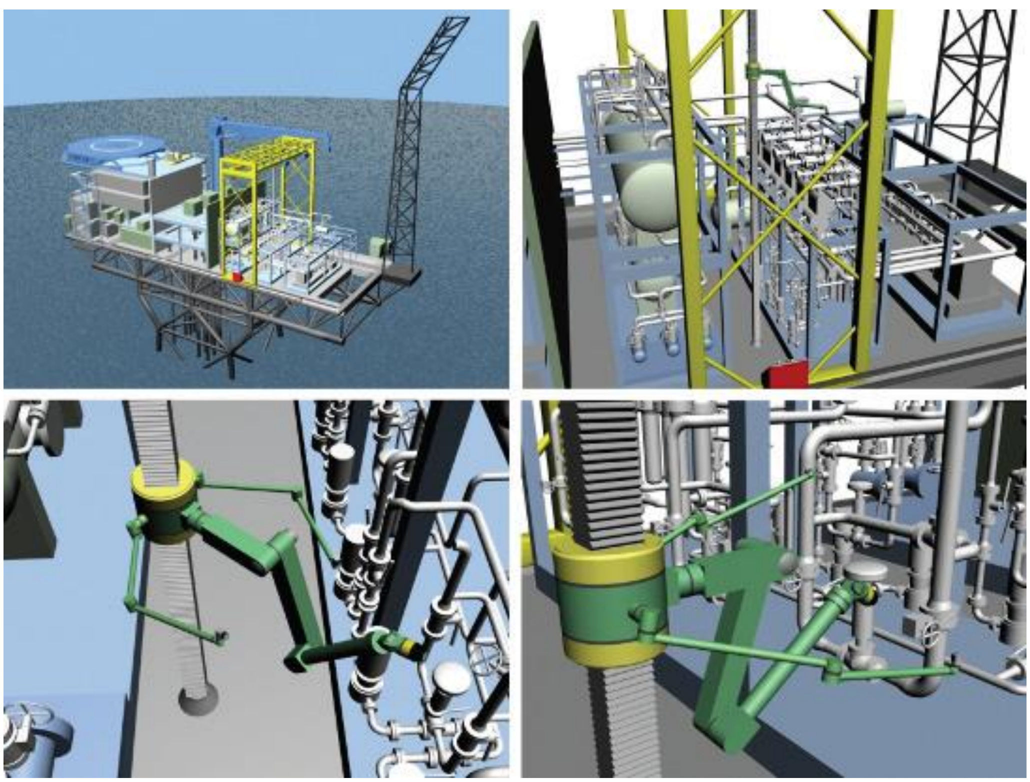


Figure 6-3 Illustration of the unmanned platform concept [168].

The synchronization concept is somewhat similar to the task of tracking a reference in time. However, instead of following a desired precomputed trajectory, the reference is the motion of the physical manipulators. We use the concept of cooperative control for synchronization of multi-robot manipulators. In this synchronization scheme a multi-robot system consisting of p fully actuated robot manipulators with n rigid joints. Then a common desired trajectory $q_d, \dot{q}_d$ for all the robots is supplied. None of the n robots are assigned as leader, and all of them work together to achieve synchronization and trajectory tracking. The dynamic model of each robot manipulator is assumed to be known.

The goal of cooperative control scheme is to design control law τ_i for the all the robots in the system, such that the joint positions and velocities $q_i, \dot{q}_i$ $(i = 1, \ldots\ldots p)$ of the robots are synchronized with both the common desired trajectory $q_d, \dot{q}_d$ and to the joints of the other robots $q_j, \dot{q}_j$ $(j = 1, \ldots\ldots\ldots p, j \neq i)$. The concept is illustrated in Figure. 6-4.

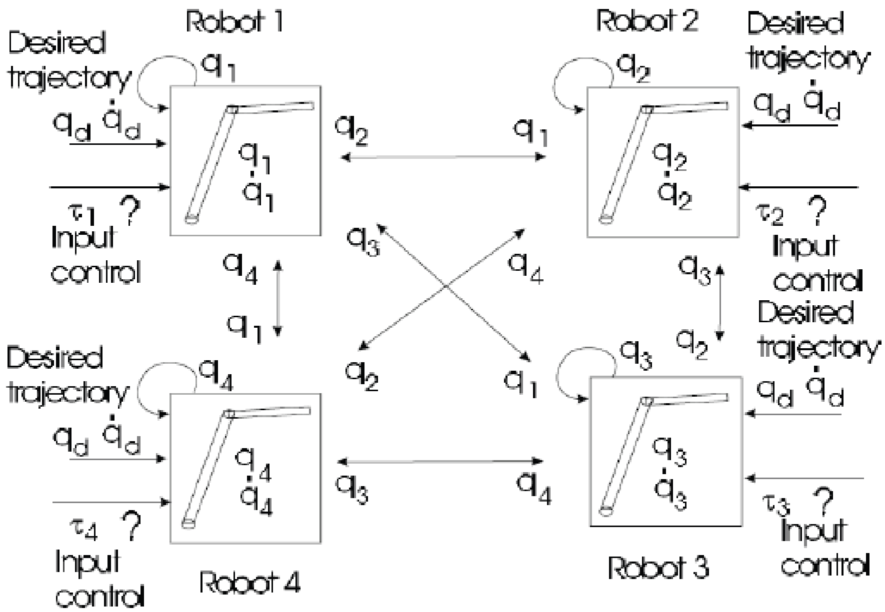


Figure 6-4 Cooperative control scheme (CCS) [169].

Our interest in this work will be mostly centered on cooperative synchronization occurring in multi-robot manipulator systems. Since the systems are nonlinear in nature, they have sensitivity dependence on initial conditions, the possibility of synchronizing two or more of such systems starting from different initial conditions is certainly an interesting feature with many useful applications. Interestingly, synchronization is most often accompanied by control in order to make it perfect for the desired application. In our work, we are going to use cooperative synchronization of multi-robot manipulators based on robust adaptive control scheme with same as well as different initial conditions.

6.5 Dynamic Model of Robot Manipulators

Considering the Euler-Lagrange formalism [167], [168] the dynamic model of the ith robot manipulator is given by

$$M_i(q_i)\ddot{q}_i + h_i(q_i,\dot{q}_i) = \tau_i + J_i^T(q_i)F_{ext(i)}(t),\ i = 1,\dots,p \tag{6.14}$$

Where p stands for the number of robot manipulator joints, and assuming that all the joints are rotational and fully actuated. In the above equation, $M_i(q_i), h_i(q_i,\dot{q}_i)$ and $F_{ext(i)}(t)$ are unknown and this values can be calculated as:

Property 6.1: The inertia matrix is decomposed as

$$M_i(q_i) = M_{0i}(q_i) + M_{\Delta i}(q_i) \tag{6.15}$$

In which $M_{0i}(q_i)$ denotes the known nominal part, and $M_{\Delta i}(q_i)$ is a norm-bounded unmodelled perturbation with bounded time derivative, i.e., $\|M_{\Delta i}(q_i)\| \le \alpha_{1i}$ and $\|\dot{M}_{\Delta i}(q_i)\| \le \alpha_{2i}$ where α_{1i}, α_{2i} are unknown constants.

Property 6.2: Due to the variations in such parameters included in matrix $h_i(q_i,\dot{q}_i)$ it can be expressed in a general form as

$$h_i(q_i,\dot{q}_i) = h_{0i}(q_i,\dot{q}_i) + \Phi_i(q_i,\dot{q}_i)\Theta_i + h_{\Delta i}(q_i,\dot{q}_i) \tag{6.16}$$

Where, $h_{0i}(q_i,\dot{q}_i)$ is a known vector, Θ_i represents the vector of uncertain parameters, $\Phi_i(q_i,\dot{q}_i)$ is a dimensionally compatible matrix, and $h_{\Delta i}(q_i,\dot{q}_i)$ is the vector of unstructured uncertainties such that $\|h_{\Delta i}(q_i,\dot{q}_i)\| \le \alpha_{3i}$, where α_{3i} is an uncertain parameter.

Property 6.3: In general, the external force disturbance $F_{ext(i)}(t)$ is time-varying, with unknown bound, i.e., $\|F_{ext(i)}\| \le \eta_i$ where η_i is unknown.

Remark 6.1: From the properties 6.1-6.3, we almost include all kinds of disturbances and uncertainties occurred due to numerous applications of robot manipulators in different conditions, are taken into account, without any pre-assumption on the periodicity or the bound of such perturbations.

Remark 6.2: Unlike the most previous works [107], [160]–[163], [169], [170] the skew symmetric property is not required here.

The basic objective here is to track the twice differentiable desired trajectory $q_d(t)$ in the presence of uncertainties, unknown time-varying parameters and external force disturbances.

6.6 Synchronized Robust Adaptive Controller Design

If the full state of all the robot manipulators in the multi-composed system is available, then we can propose robust adaptive mutual synchronization controller τ_i for the ith robot manipulator, $i = 1, \ldots\ldots, p$, for the robotic system (6.14) ensuring robustness with respect to various kinds of model uncertainties and external disturbances.

As a preliminary step to develop the controller, define the synchronization error as

$$\epsilon_i = q_{ri}(t) - q_i(t) \tag{6.17}$$

In order to guarantee the synchronized interactions between robot manipulators and generate synchronous behavior, define reference signal as [171]

$$q_{ri} = q_d - \sum_{j=1, j\neq i}^{p} K_{cp_i,j}(q_i - q_j) \tag{6.18}$$

$$\dot{q}_{ri} = \dot{q}_d - \sum_{j=1, j\neq i}^{p} K_{cv\_i,j}(\dot{q}_i - \dot{q}_j) \tag{6.19}$$

$$\ddot{q}_{ri} = \ddot{q}_d - \sum_{j=1, j\neq i}^{p} K_{ca\_i,j}(\ddot{q}_i - \ddot{q}_j) \tag{6.20}$$

Where $K_{cp_i,j}, K_{cv\_i,j}, K_{ca\_i,j} \in R^{n\times n}$, $i = 1, \ldots\ldots, p$, are position, velocity and acceleration positive semi-definite diagonal gain matrices respectively, these matrices define the interactions between the robot manipulators in the system. With the above defined synchronous error and reference signal, these manipulators are made to follow the synchronized desired trajectory $q_d(t)$. From the equations of $q_{ri}, \dot{q}_{ri},$ and $\ddot{q}_{ri}$ the second term corresponds to the "feedback" of the errors calculated between ith manipulator and the other manipulators in the system. From (6.18)-(6.20), it is clear that given robot manipulators not only tracks down the desired path but also they keep in track with the mutual distances between each of them, thus carrying out two tasks together i.e., tracking the desired trajectory as well as minimizing the distance between them.

The two metric functions are defined as [168]

$$S_i(t) = \dot{\epsilon}_i\ (t) + \wedge \epsilon_i\ (t) \tag{6.21}$$

$$S_{ri}(t) = \ddot{q}_{ri}(t) + \wedge \dot{\epsilon}_i\ (t) \tag{6.22}$$

Where $\wedge$ is a positive definite matrix, and the decaying rate of tracking error depend on it.

Remark 6.3: For simplicity it is assumed that for all $i = 1, \ldots\ldots, p$ the coupling gains $K_{cp_i,j}, K_{cv\_i,j}, K_{ca\_i,j}$ satisfy $K_{cp_i,j} = K_{cv\_i,j} = K_{ca\_i,j} = K_{i,j}$.

Based on the notations used in properties 6.1 and 6.2, define positive parameter $\alpha_i = max\{\alpha_{1i}, \alpha_{2i}, \alpha_{3i}\}$. The robust adaptive mutual synchronization controller τ_i is proposed here as

$$\tau_i = h_{0i}(q_i, \dot{q}_i) + M_{0i}(q_i)S_{ri} + \left(\frac{1}{2}\dot{M}_{0i}(q_i) + 2K_{p,i}\right)S_i + \tau_{a1i} + \tau_{a2i} - K_{p,i}\,\dot{\epsilon}_i - K_{d,i}\,\epsilon_i \tag{6.23}$$

Where $K_{p,i}, K_{d,i}$ are positive definite matrices, with τ_{a1i} deals with unstructured uncertainties, and τ_{a2i}, tackles the unknown parameters and disturbances, are adaptive sub controllers to be designed. Also, we have $K_{p,i} = \wedge K_{d,i}$.

Theorem 6.1: For uncertain robotic systems, described by (6.14) with properties 6.1-6.3, the smooth bounded reference trajectory $q_{ri}(t)$ is given. Consider the control law (6.23) with

$$\tau_{a1i} = \hat{\alpha}_i S_i \left(\frac{1}{2} + \hat{\alpha}_i \frac{1}{\|S_i\|\hat{\alpha}_i + \frac{1}{3}\delta e^{-\sigma t}} + \hat{\alpha}_i \frac{S_{ri}^T S_{ri}}{\|S_{ri}\|\|S_i\|\hat{\alpha}_i + \frac{1}{3}\delta e^{-\sigma t}}\right) \tag{6.24}$$

$$\tau_{a2i} = \Phi_i(q_i, \dot{q}_i)\widehat{\Theta}_i + \hat{\eta}_i^2 \frac{J_i^T J_i S_i}{\left\|S_i^T J_i^T\right\|\hat{\eta}_i + \frac{1}{3}\delta e^{-\sigma t}} \tag{6.25}$$

Where $\hat{\alpha}_i, \widehat{\Theta}_i$ and $\hat{\eta}_i$ are updated with adaptation laws with δ and σ are very small constants specified by the designer to avoid discontinuity.

The adaptation mechanism is given as

$$\dot{\hat{\alpha}}_i = \gamma_\alpha \left((\|S_{ri}\| + 1)\|S_i\| + \frac{1}{2} S_i^T S_i \right) \tag{6.26}$$

$$\dot{\hat{\eta}}_i = \gamma_\eta \left\| S_i^T J_i^T \right\| \tag{6.27}$$

$$\dot{\widehat{\Theta}}_i = \Gamma \Phi_i^T (q_i, \dot{q}_i) S_i \tag{6.28}$$

In which γ_α and γ_η are adaptation gains and $\Gamma = \Gamma^T > 0$ is adaptation matrix. The synchronized robust adaptive controller formed by (6.23)-(6.28), ensures the convergence of tracking error, despite the perturbations.

Proof: Choose the Lyapunov function

$$U_i\left(\epsilon_i, \dot{\epsilon}_i, \tilde{\alpha}_i, \tilde{\eta}_i, \widetilde{\Theta}_i\right) = \sum_{i=1}^{p} V_i(\epsilon_i, \dot{\epsilon}_i) + \frac{1}{2\gamma_\alpha} \tilde{\alpha}_i^2 + \frac{1}{2\gamma_\eta} \tilde{\eta}_i^2 + \frac{1}{2} \tilde{\Theta}_i^T \Gamma^{-1} \tilde{\Theta}_i \tag{6.29}$$

With

$$V_i(\epsilon_i, \dot{\epsilon}_i) = \sum_{i=1}^{p} \epsilon_i^T \; K_{d,i} \; \epsilon_i + \frac{1}{2} S_i^T M_i S_i \tag{6.30}$$

Where $\tilde{\alpha}_i = \alpha_i - \hat{\alpha}_i$, $\tilde{\eta}_i = \eta_i - \hat{\eta}_i$ and $\widetilde{\Theta}_i = \Theta_i - \widehat{\Theta}_i$ denote the parameter estimation errors. The time derivative of (6.30) is

$$\dot{V}_i(\epsilon_i, \dot{\epsilon}_i) = \sum_{i=1}^{p} \dot{\epsilon}_i^T \; K_{d,i} \; \epsilon_i + \epsilon_i^T \; K_{d,i} \; \dot{\epsilon}_i + \frac{1}{2} \dot{S}_i^T M_i S_i + \frac{1}{2} S_i^T \dot{M}_i S_i + \frac{1}{2} S_i^T M_i \dot{S}_i \tag{6.31}$$

$$\dot{V}_i(\epsilon_i, \dot{\epsilon}_i) = \sum_{i=1}^{p} 2 \, \epsilon_i^T \; K_{d,i} \; \dot{\epsilon}_i + S_i^T M_i \dot{S}_i + \frac{1}{2} S_i^T \dot{M}_i S_i \tag{6.32}$$

$$\dot{V}_i(\epsilon_i, \dot{\epsilon}_i) = \sum_{i=1}^{p} 2\, \epsilon_i^T\, K_{d,i}\, \dot{\epsilon}_i + S_i^T (M_i \ddot{q}_{ri} - M_i \ddot{q}_i + M_i \wedge \dot{\epsilon}_i) + \frac{1}{2} S_i^T \dot{M}_i S_i \tag{6.33}$$

$$\dot{V}_i(\epsilon_i, \dot{\epsilon}_i) = \sum_{i=1}^{p} 2\, \epsilon_i^T\, K_{d,i}\, \dot{\epsilon}_i + S_i^T \left(M_i \ddot{q}_{ri} + h_i(q_i, \dot{q}_i) - \tau_i - J_i^T(q_i) F_{ext(i)} + M_i \wedge \dot{\epsilon}_i + \frac{1}{2} \dot{M}_i S_i \right) \tag{6.34}$$

Replacing the τ_i with the one proposed earlier

$$\begin{aligned} \dot{V}_i(\epsilon_i, \dot{\epsilon}_i) = \sum_{i=1}^{p} 2\, \epsilon_i^T\, K_{d,i}\, \dot{\epsilon}_i &+ S_i^T \Big(M_i \ddot{q}_{ri} + h_i(q_i, \dot{q}_i) - h_{0i}(q_i, \dot{q}_i) - M_{0i}(q_i) S_{ri} \\ &- \left(\frac{1}{2} \dot{M}_{0i}(q_i) + 2K_{p,i} \right) S_i - \tau_{a1i} - \tau_{a2i} + K_{p,i}\, \dot{\epsilon}_i \\ &+ K_{d,i}\, \epsilon_i - J_i^T(q_i) F_{ext(i)} + M_i \wedge \dot{\epsilon}_i + \frac{1}{2} \dot{M}_i S_i \Big) \end{aligned} \tag{6.35}$$

Replacing the uncertain matrix $M_i(q_i)$ and unknown vector $h_i(q_i, \dot{q}_i)$ from (6.15) and (6.16) respectively.

$$\begin{aligned} \dot{V}_i(\epsilon_i, \dot{\epsilon}_i) = \sum_{i=1}^{p} 2\, \epsilon_i^T\, K_{d,i}\, \dot{\epsilon}_i - S_i^T K_{p,i} S_i &+ S_i^T \Big(M_{\Delta i}(\ddot{q}_{ri} + \dot{\epsilon}_i) + \frac{1}{2} \dot{M}_{\Delta i} S_i + \Phi_i(q_i, \dot{q}_i) \Theta_i \\ &+ h_{\Delta i}(q_i, \dot{q}_i) - J_i^T(q_i) F_{ext(i)} - \tau_{a1i} - \tau_{a2i} \Big) \end{aligned} \tag{6.36}$$

Now, taking into account properties 6.1-6.3 implies that

$$\dot{V}_i(\in_i, \dot{\in}_i) \leq \sum_{i=1}^{p} -\in_i^T \ K_{d,i} \in_i -\dot{\in}_i^T \ K_{d,i} \ \dot{\in}_i + \alpha_{1i}\|S_{ri}\|\|S_i\| + \frac{1}{2}\alpha_{2i}S_i^T S_i + \alpha_{3i}\|S_i\| + S_i^T \Phi_i(q_i, \dot{q}_i)\Theta_i - S_i^T J_i^T F_{ext(i)} - S_i^T \tau_{a1i} - S_i^T \tau_{a2i} \tag{6.37}$$

$$\dot{V}_i(\in_i, \dot{\in}_i) \leq \sum_{i=1}^{p} -\in_i^T \ K_{d,i} \in_i -\dot{\in}_i^T \ K_{d,i} \ \dot{\in}_i + \alpha_i \left(\frac{1}{2}S_i^T S_i + \|S_{ri}\|\|S_i\| + \|S_i\|\right) + S_i^T \Phi_i(q_i, \dot{q}_i)\Theta_i + \eta_i \left\|S_i^T J_i^T\right\| - S_i^T \tau_{a1i} - S_i^T \tau_{a2i} \tag{6.38}$$

By the sub-controllers (6.24) and (6.25), we obtain

$$-S_i^T \tau_{a1i} \leq -\frac{1}{2}\hat{\alpha}_i S_i^T S_i - \hat{\alpha}_i\|S_i\| - \hat{\alpha}_i\|S_{ri}\|\|S_i\| + \frac{2}{3}\delta e^{-\sigma t} \tag{6.39}$$

$$-S_i^T \tau_{a2i} \leq -S_i^T \Phi_i(q_i, \dot{q}_i)\widehat{\Theta}_i - \hat{\eta}_i\left\|S_i^T J_i^T\right\| + \frac{1}{3}\delta e^{-\sigma t} \tag{6.40}$$

Incorporating the inequalities (6.39) and (6.40) into (6.38), gives

$$\dot{V}_i(\in_i, \dot{\in}_i) \leq \sum_{i=1}^{p} -\in_i^T \ K_{d,i} \in_i -\dot{\in}_i^T \ K_{d,i} \ \dot{\in}_i + \tilde{\alpha}_i \left(\frac{1}{2}S_i^T S_i + \|S_{ri}\|\|S_i\| + \|S_i\|\right) + \tilde{\eta}_i\left\|S_i^T J_i^T\right\| + S_i^T \Phi_i(q_i, \dot{q}_i)\widetilde{\Theta}_i \tag{6.41}$$

On the other hand, the time derivative of (6.29) is calculated as

$$\dot{U}_i\left(\in_i, \dot{\in}_i, \tilde{\alpha}_i, \tilde{\eta}_i, \widetilde{\Theta}_i\right) = \sum_{i=1}^{p} \dot{V}_i(\in_i, \dot{\in}_i) - \frac{1}{\gamma_\alpha}\tilde{\alpha}_i\dot{\hat{\alpha}}_i - \frac{1}{\gamma_\eta}\tilde{\eta}_i\dot{\hat{\eta}}_i - \dot{\widehat{\Theta}}_i^T \Gamma^{-1}\tilde{\Theta}_i \tag{6.42}$$

Using (6.41) and update laws (6.26)-(6.28) in (6.42) yields

$$\dot{U}_i\left(\in_i, \dot{\in}_i, \tilde{\alpha}_i, \tilde{\eta}_i, \widetilde{\Theta}_i\right) \leq \sum_{i=1}^{p} -\in_i^T \ K_{d,i} \in_i -\dot{\in}_i^T \ K_{d,i} \ \dot{\in}_i + \delta e^{-\sigma t} \tag{6.43}$$

In order to use the Barbalat's lemma in the proof procedure, it is shown here that $\in_i\ (t)$ and $\dot{\in}_i\ (t)$ are bounded and $\in_i\ (t)$ is square-integrable.

By (6.43) one can conclude

$$\dot{U}_i\big(\in_i, \dot{\in}_i, \tilde{\alpha}_i, \tilde{\eta}_i, \widetilde{\Theta}_i\big) \le \sum_{i=1}^{p} -\in_i^T\ K_{d,i}\ \in_i + \delta e^{-\sigma t} \tag{6.44}$$

Which gives three consequents as follows

1. Integrating the inequality (6.44) from $t = 0$ to $t = T$ yields

$$\int_0^T \|\in_i\ (t)\|_K^2 dt + V_i\left(\in_i\ (T), \dot{\in}_i\ (T), \tilde{\alpha}_i(T), \tilde{\eta}_i(T), \widetilde{\Theta}_i(T)\right) \le \left(\in_i\ (0), \dot{\in}_i\ (0), \tilde{\alpha}_i(0), \tilde{\eta}_i(0), \widetilde{\Theta}_i(0)\right) + \frac{\delta}{\sigma}(1 - e^{-\sigma T}) \tag{6.45}$$

For all $0 \le T < \infty$. This implies that $e_i(t)$ is square integrable

2. By (6.44), $\dot{U}_i$ can be bounded as $\dot{U}_i \le -\lambda_K \|\in_i\|^2 + \delta$ where λ_K is minimum Eigen value of K. choosing $\lambda_K > \frac{\delta}{\varepsilon^2}$ for any $\varepsilon > 0$, there exists a $K > 0$, such that $\dot{U}_i \le -\kappa\|\in_i\|^2 < 0$ for all $\|\in_i\| < \varepsilon$, for all $t \ge T$, and the boundedness of synchronization error $\in_i\ (t)$ is guaranteed.

Taking the consequents (1)-(2) into account, the Barbalat's lemma guarantees the boundedness of all the closed-loop signals and the convergence of synchronization error $\in_i\ (t)$ despite the system uncertainties and external force disturbances.

Remark 6.4: The exponential term, incorporated in the sub-controllers (6.24) and (6.25), are to avoid chattering and discontinuity of the control input, without violating the convergence property of the synchronization error of closed loop stability.

6.7 Time Varying Observer Design

In this section, we develop an output feedback tracking control scheme with time invariant and time-varying observer gains for a two link manipulator whose dynamics are given in (6.14). The observer design is based only on measurement of position, using $x_{1i} = q_i \in R^n$ and $x_{2i} = \dot{q}_i \in R^n$, now the dynamics of the robot manipulator can be rewritten with change of variable in state space form as [172]:

$$\dot{x}_{1i} = x_{2i} \tag{6.46}$$

$$\dot{x}_{2i} = M_i^{-1}(x_{1i})\left(\tau_i + J_i^T(x_1)\, F_{ext(i)}(t) - h_i(x_{1i}, x_{2i})\right) \tag{6.47}$$

$$y_i = x_{1i} \tag{6.48}$$

Based on (6.46)- (6.48), the time-varying observer is designed as:

$$\dot{\hat{x}}_{1i} = \hat{x}_{2i} + L_{1i}(x_{1i} - \hat{x}_{1i}) + L_{2i}(x_{2i} - \hat{x}_{2i}) \tag{6.49}$$

$$\begin{aligned}\dot{\hat{x}}_{2i} = M_{0i}^{-1}(\hat{x}_{1i})\big(&-h_i(\hat{x}_{1i}, \hat{x}_{2i}) + \tau_i + L_{3i}(x_{1i} - \hat{x}_{1i}) \\ &+ L_{4i}(x_{2i} - \hat{x}_{2i})\big)\end{aligned} \tag{6.50}$$

Where, $L_{1i}, L_{2i}, L_{3i}, L_{4i}$ are the time varying correction terms & $\hat{x}_{1i}, \hat{x}_{2i}$ are the estimates of x_{1i}, x_{2i} and the overall diagram for the proposed scheme is shown in Figure. 6-5.

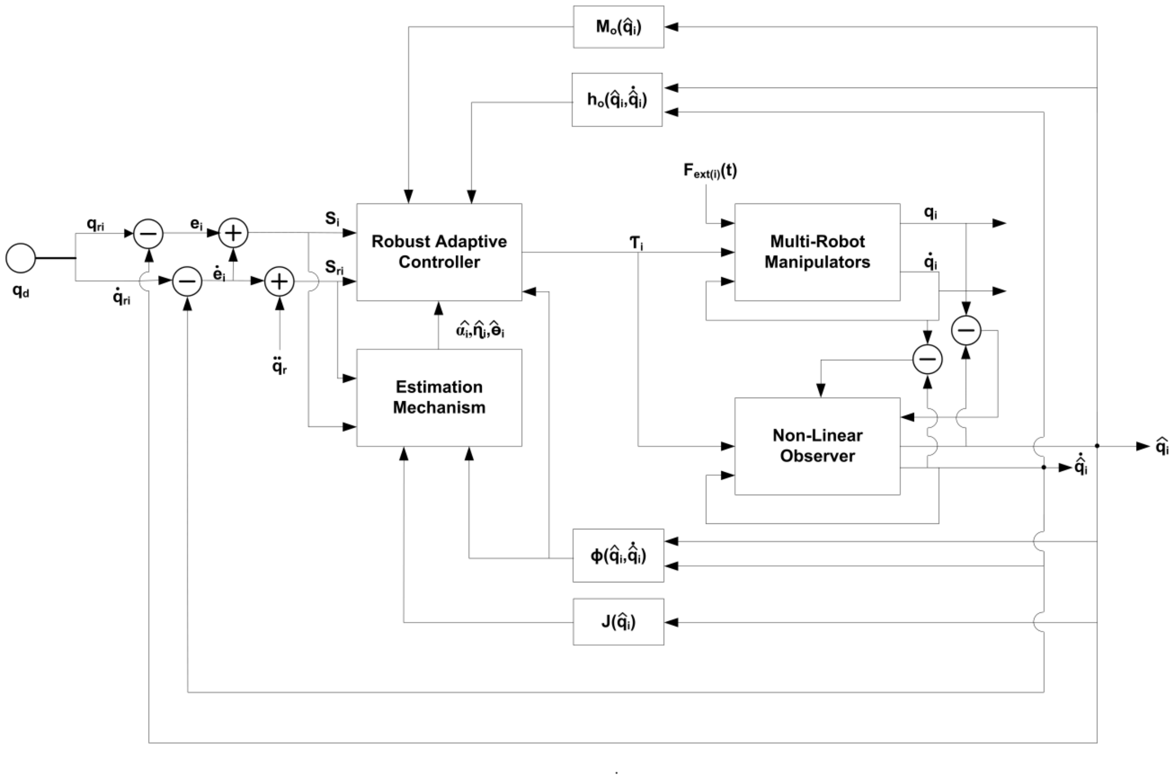


Figure 6-5 The overall diagram of the proposed scheme.

As an initial step in design define the tracking errors as

$$\dot{e}_{1i} = e_{2i} - L_{1i}e_{1i} - L_{2i}e_{2i} \tag{6.51}$$

$$\dot{e}_{2i} = -L_{3i}e_{1i} - L_{4i}e_{2i} - \zeta\varphi_i(e_{1i}, e_{2i}) + w_i(t) \tag{6.52}$$

where, $e_{1i} = x_{1i} - \hat{x}_{1i}$, $e_{2i} = x_{2i} - \hat{x}_{2i}$ are error estimates, $w_i(t)$=external force disturbance, ζ=unkown vector and $\varphi_i(e_{1i}, e_{2i})$=known nonlinear function.

Proof: Let us consider Lyapunov candidate function as

$$V_i(x_i) = \sum_{i=1}^{p} \frac{1}{2} e_{1i}^T e_{1i} + \frac{1}{2\lambda_1^4}(e_{2i} + \lambda_1(t)e_{1i})^T(e_{2i} + \lambda_1(t)e_{1i}) + \frac{1}{2}\zeta_{1i}^T\zeta_{1i} \tag{6.53}$$

With $e_{1i} = x_{1i} - \hat{x}_{1i}$,$e_{2i} = x_{2i} - \hat{x}_{2i}$ denotes the estimation errors and $\lambda_1(t)$is time-varying design parameter selected by the designer as

$$\lambda_1(t) = \lambda_{max}(1 - e^{-\varsigma t}) \tag{6.54}$$

$$\dot{\lambda}_1(t) = -\lambda_{max}(\varsigma(1 - e^{-\varsigma t}) - \varsigma) \tag{6.55}$$

$$\dot{\lambda}_1(t) = -\varsigma\lambda_1 + \varsigma\lambda_{max} \tag{6.56}$$

Let

$$V_{1i}(x_i) = \sum_{i=1}^{p} \frac{1}{2} e_{1i}^T e_{1i} \tag{6.57}$$

The time derivative of the above equation is

$$\dot{V}_{1i}(x_i) = \sum_{i=1}^{p} e_{1i}^T \dot{e}_{1i} \tag{6.58}$$

Substitute the value of $\dot{e}_1$ in the above equation, we get

$$\dot{V}_{1i}(x_i) = \sum_{i=1}^{p} e_{1i}^T (e_{2i} - L_{1i}e_{1i} - L_{2i}e_{2i}) \tag{6.59}$$

For the term $e_{1i}^T e_{2i}$ we use young's inequality with $\lambda > 0$, λ is a design parameter also selected by the designer.

We have $e_{1i}^T e_{2i} \leq \frac{\lambda e_{1i}^T e_{1i}}{2} + \frac{e_{2i}^T e_{2i}}{2\lambda}$ substitute in (6.59) we have

$$\dot{V}_{1i}(x_i) \leq \sum_{i=1}^{p} -e_{1i}^T \left(L_{1i} - \frac{\lambda}{2}\right) e_{1i} - e_{1i}^T L_{2i} e_{2i} + \frac{e_{2i}^T e_{2i}}{2\lambda} \tag{6.60}$$

Now let

$$V_{2i}(x_i) = \sum_{i=1}^{p} \frac{1}{2\lambda_1^4(t)} \, (e_{2i} + \lambda_1(t)e_{1i})^T \, (e_{2i} + \lambda_1(t)e_{1i}) \tag{6.61}$$

$$\begin{aligned} V_{2i}(x_i) = \sum_{i=1}^{p} \frac{1}{2\lambda_1^4(t)}(e_{2i}^T e_{2i}) + \frac{1}{2\lambda_1^4(t)}(2\lambda_1(t)e_{1i}^T e_{2i}) \\ + \frac{1}{2\lambda_1^4(t)}(\lambda_1^2(t)e_{1i}^T e_{1i}) \end{aligned} \tag{6.62}$$

And the time derivative of (6.62), can be calculated as:

$$\begin{aligned} \dot{V}_{2i}(x_i) = \sum_{i=1}^{p} \frac{1}{2}\left(\frac{\lambda_1^4(t)(2e_{2i}^T \dot{e}_{2i}) - e_{2i}^T e_{2i}\left(4\lambda_1^3(t)\dot{\lambda}_1(t)\right)}{(\lambda_1^4(t))^2}\right) \\ + \left(\frac{\lambda_1^3(t)(e_{1i}^T \dot{e}_{2i} + \dot{e}_{1i} e_{2i}^T) - e_{1i}^T e_{2i}\left(3\lambda_1^2(t)\dot{\lambda}_1(t)\right)}{(\lambda_1^3(t))^2}\right) \\ + \frac{1}{2}\left(\frac{\lambda_1^2(2e_{1i}^T \dot{e}_{1i}) - e_{1i}^T e_{1i}\left(2\lambda_1 \dot{\lambda}_1\right)}{(\lambda_1^2(t))^2}\right) \end{aligned} \tag{6.63}$$

Taking like terms together

$$\begin{aligned} \dot{V}_{2i}(x_i) = \sum_{i=1}^{p} \left(\frac{e_{2i}^T}{\lambda_1^4(t)} + \frac{e_{1i}^T}{\lambda_1^3(t)}\right)\dot{e}_{2i} + \left(\frac{e_{2i}^T}{\lambda_1^3(t)} + \frac{e_{1i}^T}{\lambda_1^2(t)}\right)\dot{e}_{1i} \\ - \left(\frac{2e_{2i}^T e_{2i}}{\lambda_1^5(t)} + \frac{3e_{1i}^T e_{2i}}{\lambda_1^4(t)} + \frac{e_{1i}^T e_{1i}}{\lambda_1^3(t)}\right)\dot{\lambda}_1(t) \end{aligned} \tag{6.64}$$

on replacing $\dot{e}_{1i}$, $\dot{e}_{2i}$, $\dot{\lambda}_1(t)$ in the above equation we have

$$\begin{aligned} \dot{V}_{2i}(x_i) = \sum_{i=1}^{p} \left(\frac{e_{2i}^T}{\lambda_1^4(t)} + \frac{e_{1i}^T}{\lambda_1^3(t)}\right)(-L_{3i}e_{1i} - L_{4i}e_{2i} - \zeta\varphi_i(e_{1i}, e_{2i}) \\ + w_i(t)\,) + \left(\frac{e_2^T}{\lambda_1^3(t)} + \frac{e_1^T}{\lambda_1^2(t)}\right)(e_{2i} - L_{1i}e_{1i} - L_{2i}e_{2i}) \\ - \left(\frac{2e_{2i}^T e_{2i}}{\lambda_1^5(t)} + \frac{3e_{1i}^T e_{2i}}{\lambda_1^4(t)} + \frac{e_{1i}^T e_{1i}}{\lambda_1^3(t)}\right)(-\varsigma\lambda_1(t) + \varsigma\lambda_{max}) \end{aligned} \tag{6.65}$$

$$
\begin{aligned}
\dot{V}_{2i}(x_i) = \sum_{i=1}^{p} & -\frac{1}{\lambda_1^4(t)} e_{2i}^T \Big(L_{3i} + \lambda_1(t) L_{4i} + \lambda_1(t) L_{1i} \\
& + \lambda_1^2(t)(L_{2i} - 1) + 3\varsigma\big(\lambda_{max} - \lambda_1(t)\big)\Big) e_{1i} \\
& - \frac{1}{\lambda_1^4(t)} e_{2i}^T \Bigg(L_{4i} + \lambda_1(t)(L_{2i} - 1) \\
& + 2\varsigma\left(\frac{\lambda_{max}}{\lambda_1(t)} - 1\right)\Bigg) e_{2i} \\
& - \frac{1}{\lambda_1^3(t)} e_{1i}^T \Big(L_{3i} + \lambda_1(t) L_1 + \varsigma\big(\lambda_{max} - \lambda_1(t)\big)\Big) e_{1i} \\
& - \left(\frac{e_{2i}^T}{\lambda_1^4(t)} + \frac{e_{1i}^T}{\lambda_1^3(t)}\right)(\zeta_i \varphi_i(e_{1i}, e_{2i}) - w_i(t))
\end{aligned}
\tag{6.66}
$$

Finally, let us select

$$
V_3(x_i) = \sum_{i=1}^{p} \frac{1}{2} \zeta_i^T \zeta_i \tag{6.67}
$$

And find the time derivative, and that is given as

$$
\dot{V}_{3i}(x_i) = \sum_{i=1}^{p} \dot{\zeta}_i^T \zeta_i \tag{6.68}
$$

Combining all the terms for

$$
\dot{V}_i(x_i) = \sum_{i=1}^{p} \dot{V}_{1i}(x_i) + \dot{V}_{2i}(x_i) + \dot{V}_{3i}(x_i) \tag{6.69}
$$

Substitute previous values in the above equation yields

$$
\begin{aligned}
\dot{V}_i(x_i) \leq \sum_{i=1}^{p} & -\frac{1}{\lambda_1^4(t)} e_{2i}^T \Big(L_{3i} + \lambda_1(t)L_{4i} + \lambda_1(t)L_{1i} + {\lambda_1}^2(t) \\
& + {\lambda_1}^2(t)(L_{2i} - 1) + {\lambda_1}^4(t)L_{2i} \\
& + 3\varsigma\big(\lambda_{max} - \lambda_1(t)\big)\Big) e_{1i} \\
& - \frac{1}{\lambda_1^4(t)} e_{2i}^T \Bigg(L_{4i} + \lambda_1(t)(L_{2i} - 1) - \frac{{\lambda_1}^4(t)}{2\lambda} \\
& + 2\varsigma\left(\frac{\lambda_{max}}{\lambda_1(t)} - 1\right)\Bigg) e_{2i} \\
& - \frac{1}{\lambda_1^3(t)} e_{1i}^T \Bigg(L_{3i} + \lambda_1(t)L_{1i} + {\lambda_1}^3(t)\left(L_{1i} - \frac{\lambda}{2}\right) \\
& + \varsigma\big(\lambda_{max} - \lambda_1(t)\big)\Bigg) e_{1i} \\
& - \zeta_i \left(\left(\frac{e_{2i}^T}{\lambda_1^4(t)} + \frac{e_{1i}^T}{\lambda_1^3(t)}\right)\varphi_i(e_1, e_2) - \dot{\zeta}_i^T\right) \\
& + \left(\frac{e_{2i}^T}{\lambda_1^4(t)} + \frac{e_{1i}^T}{\lambda_1^3(t)}\right) w_i(t)
\end{aligned}
\tag{6.70}
$$

The right hand-side of the equation will be negative with proper selection of the correction terms $L_{1i}, L_{2i}, L_{3i}, L_{4i}$ and $\lambda, \lambda_1(t)$, leaving behind only positive term containing disturbance $w_i(t)$.

Remark 6.4: The design parameters $\lambda, \lambda_1(t)$ should be selected as big as possible to make the effect of external disturbance as small as possible, thus stabilizing the system.

Remark 6.5: It is convenient to choose $L_{1i} \gg \frac{\lambda}{2}, L_{2i} \gg 1, L_{4i} \gg \lambda_1^4(t)$ so as to cancel the negative effect of design parameter terms.

Remark 6.6: selecting $\dot{\zeta}_i^T = \frac{1}{2\lambda_1^4(t)}\left(\varphi(e_{1i}, e_{2i})(e_{2i}^T + \lambda_1(t)e_{1i}^T)\right)$ makes sure that there is no effect of ζ_i on the stability of the system.

Taking the above remarks into account, the Lyapunov function guarantees the boundedness of all closed-loop signals and the convergence of tracking error despite the system uncertainties and external force disturbances.

6.8 Simulation Results

The equation of motion can be written in the form of (6.14) with [173] as;

$$M_0(q) = \begin{bmatrix} m_1 {l_1}^2 + m_2 \overline{l_2}^2 & m_2 l_2 (l_2 + l_1 \cos\ q_2) \\ m_2 l_2 (l_2 + l_1 \cos\ q_2) & m_2 {l_2}^2 \end{bmatrix} \tag{6.71}$$

$$\mathrm{h}_0 (\mathrm{q}, \dot{\mathrm{q}}) = m_2\, l_1\, l_2 \begin{bmatrix} -(2\, \dot{\mathrm{q}}_1\, \dot{\mathrm{q}}_2 + {\dot{\mathrm{q}}_2}^2)\, cos\, q_2 \\ {\dot{\mathrm{q}}_1}^2\, sin\, q_2 \end{bmatrix} + \begin{bmatrix} (m_1 + m_2)\, g\, l_1\, cos\, q_1 + m_2\, g\, l_2\, cos\, (q_1 + q_2) \\ m_2\, g\, l_2\, cos\, (q_1 + q_2) \end{bmatrix} \tag{6.72}$$

In which $\overline{l_2}^2 = {l_1}^2 + {l_1}^2 + 2\, l_1 l_2 \cos\ q_2$, and the Jacobian matrix J is given by:

$$J(q) = \begin{bmatrix} -l_1 \sin\ q_1 - l_2 \sin\ (q_1 + q_2) & -l_2 \sin\ (q_1 + q_2) \\ l_1 \cos\ q_1 + l_2 \cos\ (q_1 + q_2) & l_2 \cos\ (q_1 + q_2) \end{bmatrix} \tag{6.73}$$

$$\Phi(q, \dot{q}) = \begin{bmatrix} gl_1 cosq_1 & gl_1 cosq_1 + gl_2 \cos(q_1 + q_2) - l_1 l_2 (2\dot{q}_1 \dot{q}_2 + \dot{q}_2^2) cosq_2 \\ 0 & l_1 l_2 \dot{q}_1^2 sinq_2 + gl_2 \cos(q_1 + q_2) \end{bmatrix} \tag{6.74}$$

Let us consider the desired trajectory be $q_d(t) = [q_{d1}(t) \quad q_{d2}(t)]^T = 0.5[\sin t \quad \cos t]^T$, and the nominal parameters be $m_{10} = 2$ Kg, $m_{20} = 1$ Kg, $l_1 = l_2 = 0.5$ m and $g = 9.8 \quad m/s^2$. The initial conditions are assumed to be $q(0) = [0.5 \quad 0]^T$ and $\dot{q}(0) = [0 \quad 0]^T$.

The observer parameters are assumed to be $L_1 = 1 + \frac{\lambda}{2}, L_2 = 1 + \lambda_1, L_3 = 500, L_4 = 1 + (\lambda_1)^4$ and $\lambda = 100, \lambda_{max}$ =100, $\varsigma = 1$, the robust adaptive controllers are constructed by $K = 2I_{2\times2}, \Lambda = diag(5,8), \delta = 0.2, \sigma = 0.1$. The constant parametric uncertainty is considered in dynamical model by taking the mass parameters as $m_1 = m_{10} + \theta_{m_1}$ and $m_2 = m_{20} + \theta_{m_2}$, where $\theta_{m_i}, i = 1,2$; denotes and unknown constant assumed to be 20% of the nominal value of the simulation. The external force is evaluated by applying $F_{ext}(t) = [F_1(t) \quad F_2(t)]^T$, on end-effector at t=1 sec. The time response of applied force is depicted in Figure. 6-7.

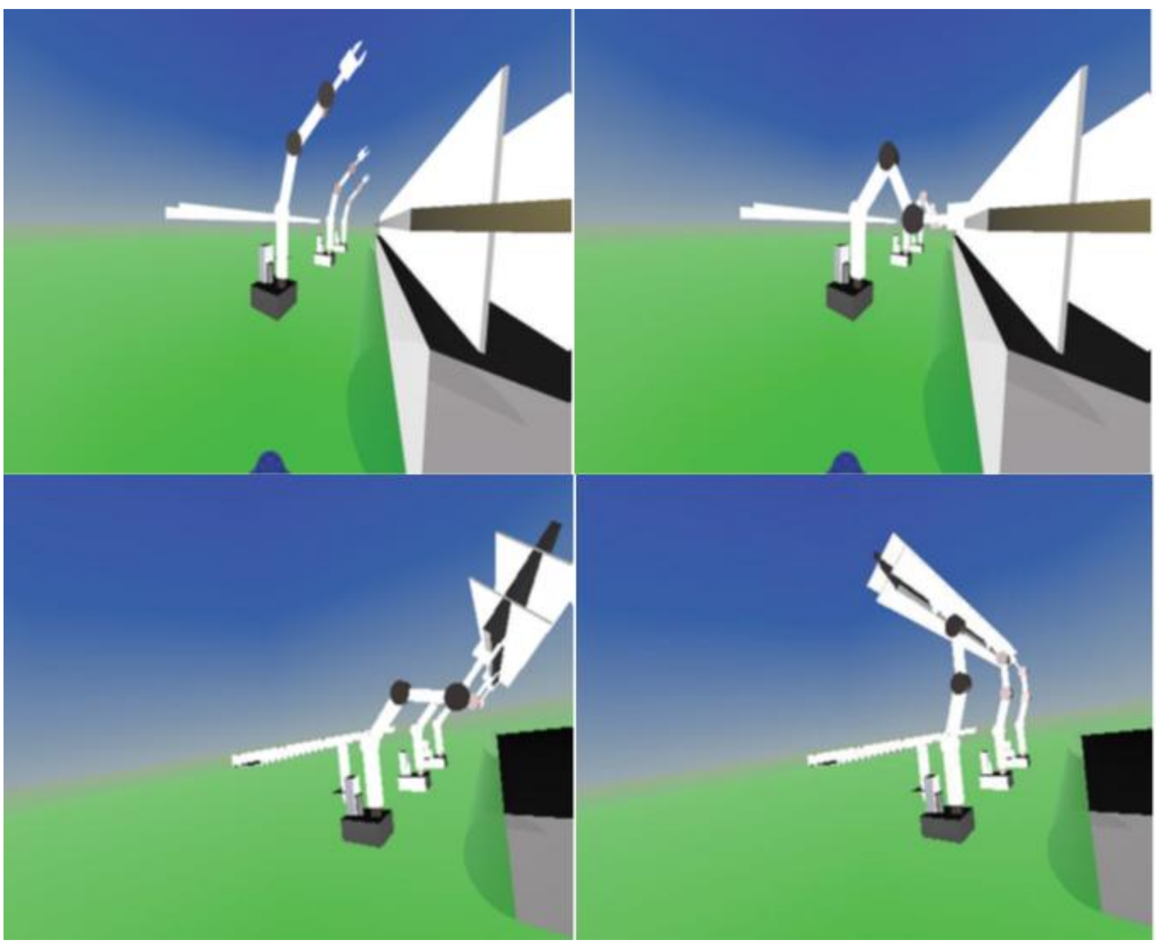


Figure 6-6 The 3D virtual environment of three robot manipulator working synchronously [20].

In order to verify the effectiveness of the proposed control algorithm, the performance of the closed – loop system is performed on identical three manipulators with two degree of freedom as depicted in Figure. 6-6. Let the manipulators have different initial conditions so that mutual synchronization phenomenon can be explained clearly, for the first manipulator let us assume 0.5 as initial value, 0.55 for second and 0.6 for third manipulator. Now let us plot the graphs for the given system.

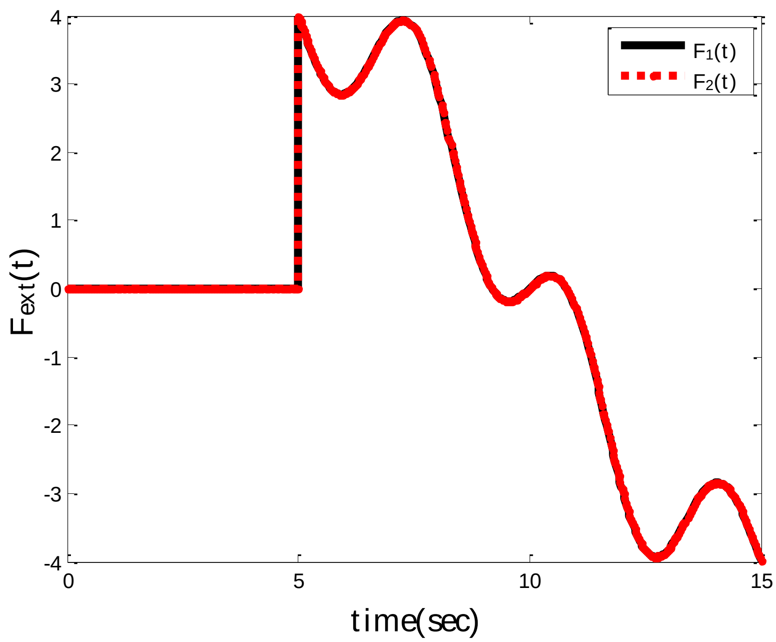


Figure 6-7 Time response of external force disturbances.

In this case we are adding external disturbance to the system and this disturbance is added at t=1 sec, this can be clearly seen from the above Figure. The main objective behind adding this external disturbance is to check the performance of the proposed controller under the influence of disturbances.

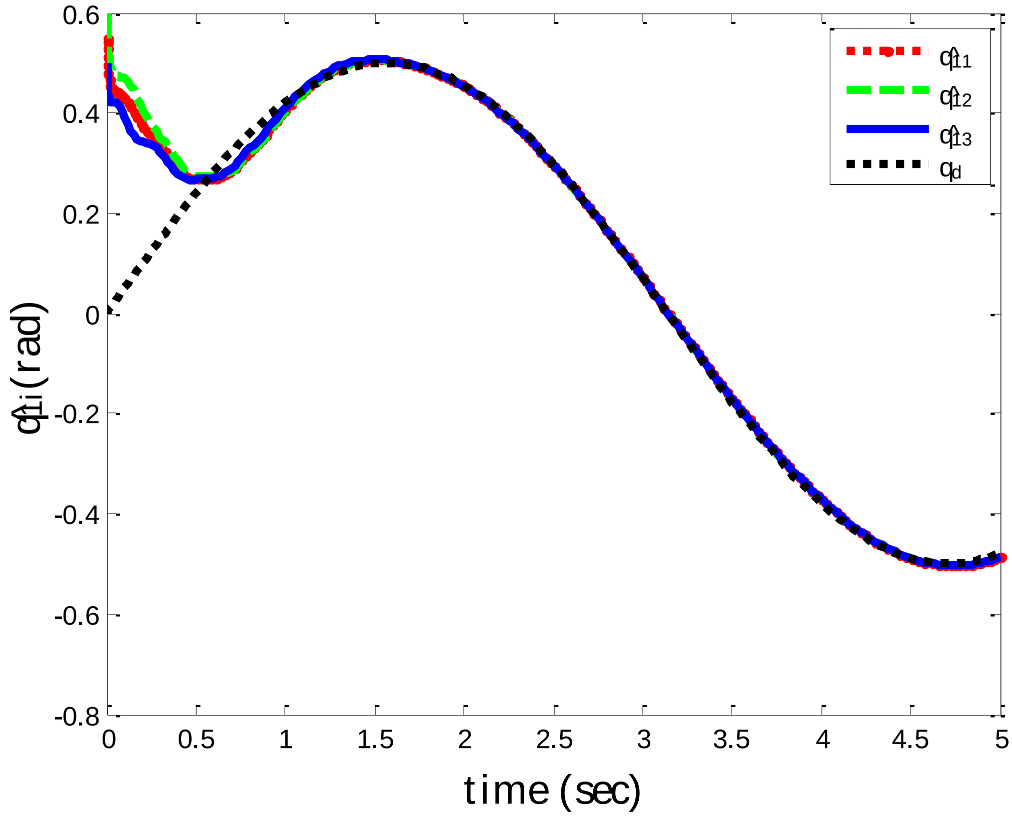


Figure 6-8 Time response of the desired trajectory to the estimated angular position 1 of multiple robot manipulators.

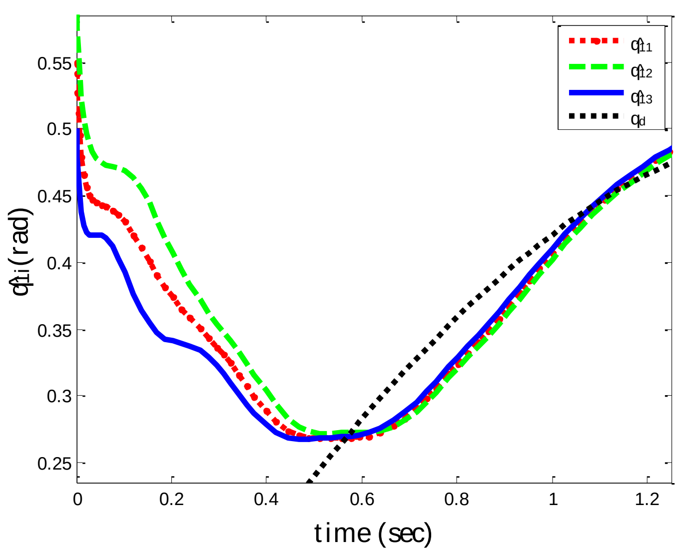


Figure 6-9 Magnified time response of the desired trajectory to the estimated angular position 1 of multiple robot manipulators.

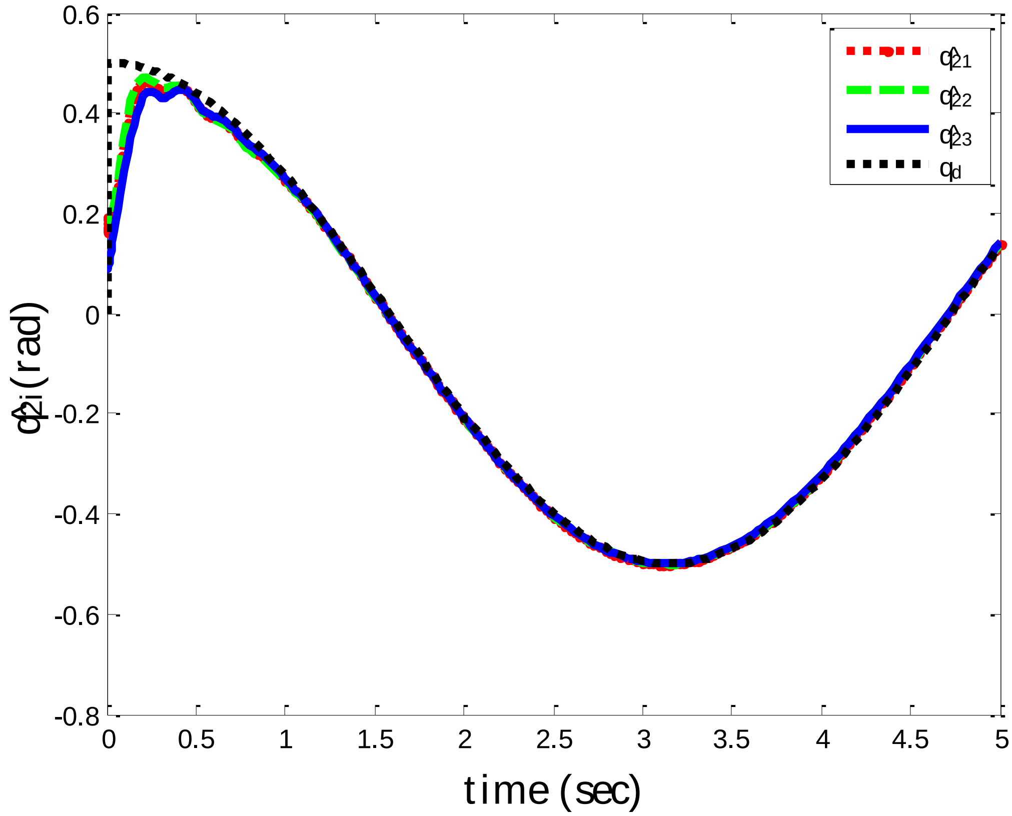


Figure 6-10 Time response of the desired trajectory to the estimated angular position 2 of multiple robot manipulators.

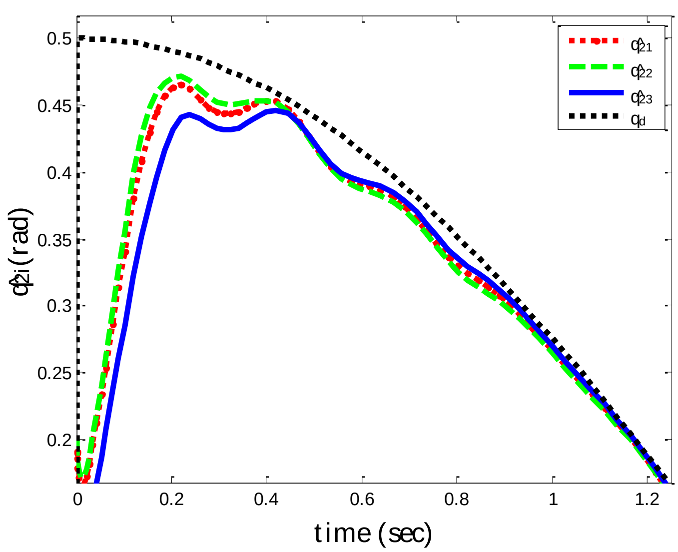


Figure 6-11 Magnified time response of the desired trajectory to the estimated angular position 2 of multiple robot manipulators.

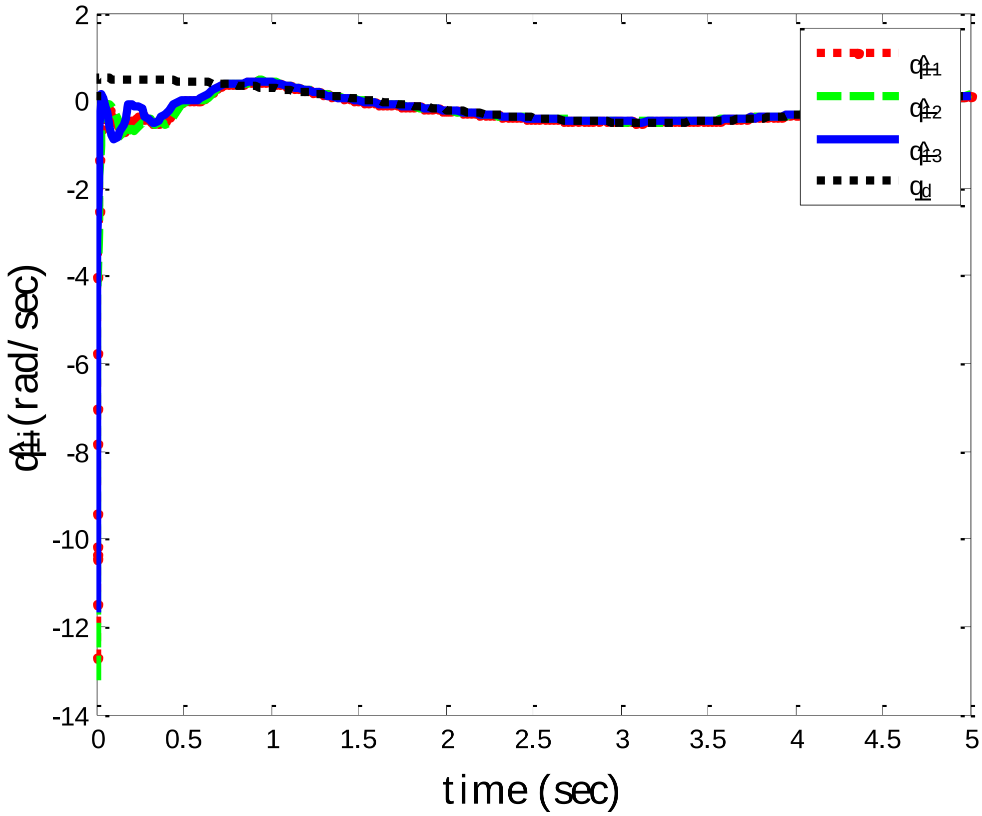


Figure 6-12 Time response of the desired trajectory and the derivative of estimated response angular velocity 1 of multiple robot manipulator.

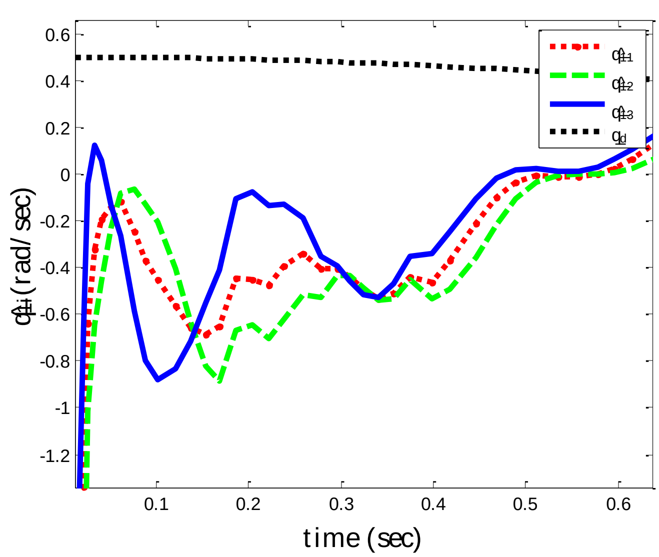


Figure 6-13 Magnified time response of the desired trajectory and the derivative of estimated response angular velocity 1 of multiple robot manipulator.

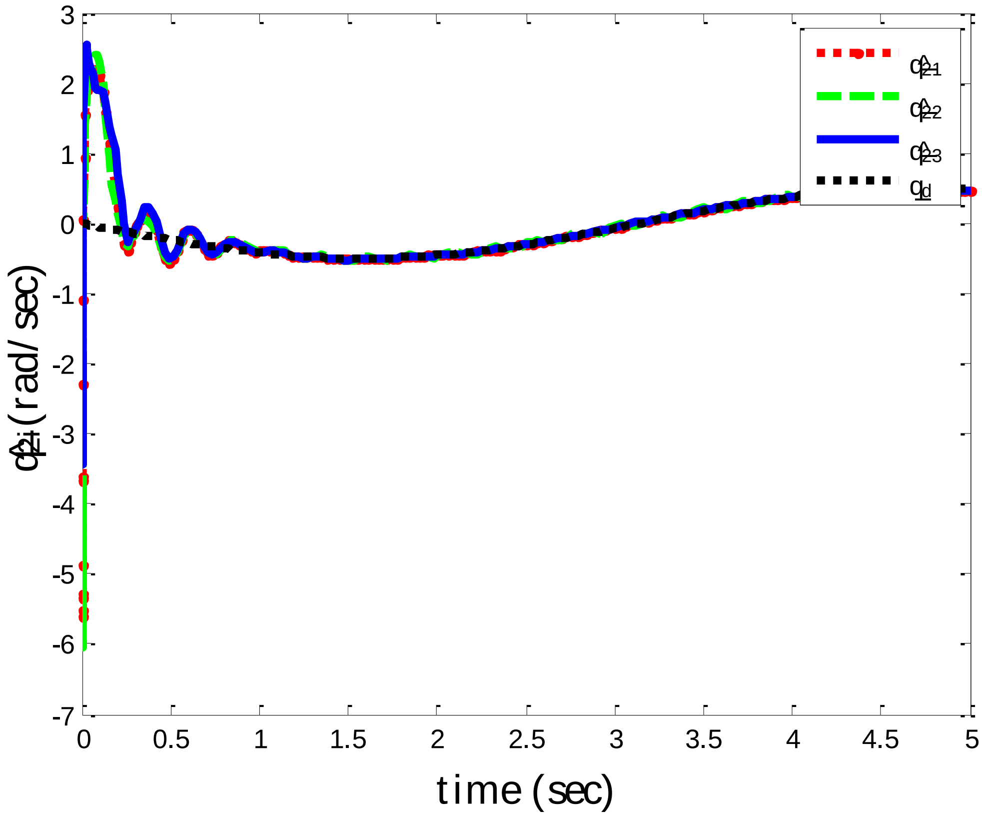


Figure 6-14 Time response of the desired trajectory and the derivative of estimated response angular velocity 2 of multiple robot manipulator.

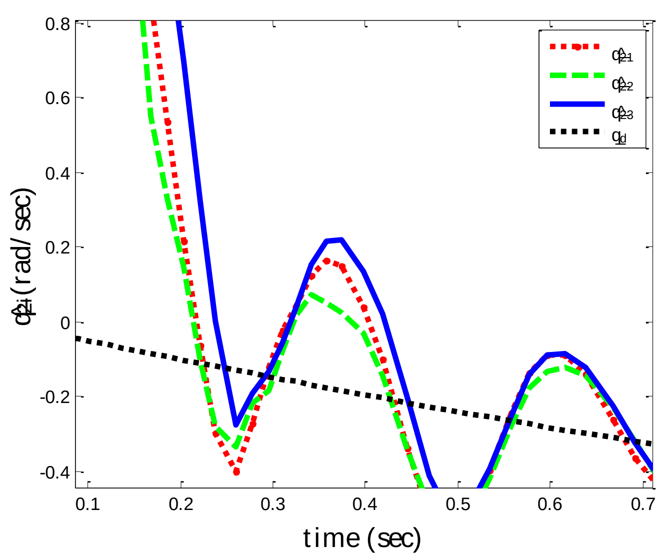


Figure 6-15 Magnified time response of the desired trajectory and the derivative of estimated response angular velocity 2 of multiple robot manipulator.

The Figure. 6-8 and 6-10, represents the angular position $\hat{q}_1(t), \hat{q}_2(t), \hat{q}_3(t)$ of the three identical robot manipulators operating synchronously. Time response of the desired trajectory to the estimated angular position of those robot are shown, and as expected from the proposed controller, the angular positions of each manipulator tracks the desired trajectory. At *t*= 1 second, the whole system is initiated with an external disturbance, the system still tracks down the desired motion with a slight variation justifying the robustness of the proposed technique.

Figure. 6-12 and 6-14, represents the angular velocities $\dot{\hat{q}}_1(t),\ \dot{\hat{q}}_2(t), \dot{\hat{q}}_3(t)$ of the three identical robot manipulators working in a synchronous environment. Time response of the desired trajectory to the estimated angular velocity of these manipulators are shown, and with the results it's clear that the proposed controller is robustly adapting itself with the change in the system conditions. We can see that the waveforms slightly varies with time and then the response is smooth. It's because of the disturbance signal introduced in the system after a short duration of time.

Figure.6.9, 6.11, 6.13, 6.15 are the magnified responses of Figure. 6.8, 6.10, 6.12, 6.14. They are drawn just to differentiate each signal more evidently and precisely. Therefore, from the above figures we can conclude that the effectiveness of the control scheme.

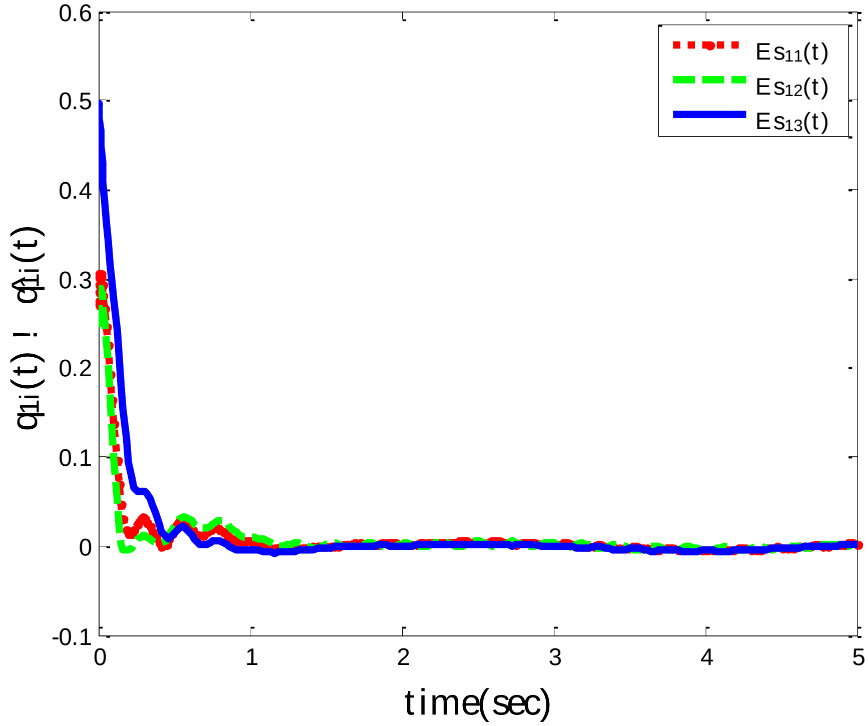


Figure 6-16 Convergence of position tracking error 1 of multiple robot manipulators.

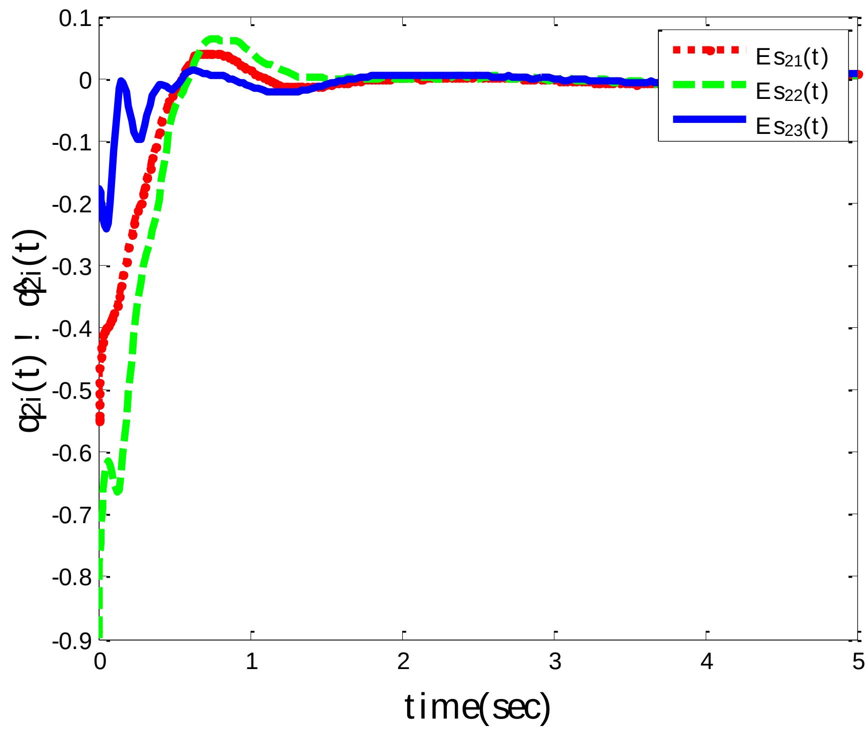


Figure 6-17 Convergence of position tracking error 2 of multiple robot manipulators.

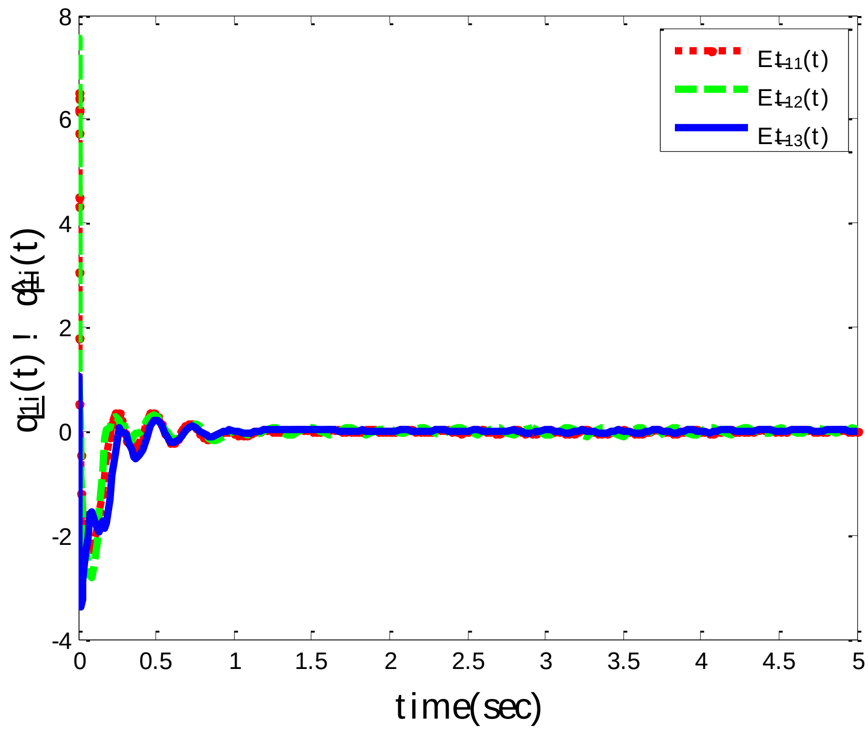


Figure 6-18 Convergence of velocity tracking error 1 for multiple robot manipulator.

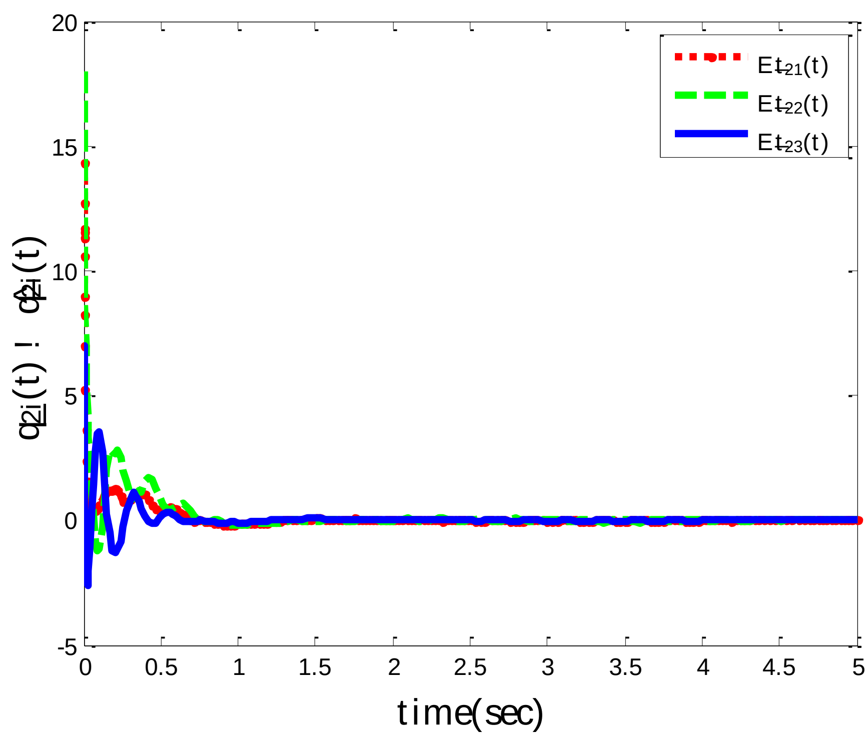


Figure 6-19 Convergence of velocity tracking error 2 for multiple robot manipulator.

Figure. 6-16, 6-17, represents the tracking and synchronization errors respectively. In any system where there are group of small sub-systems working synchronously, any small effect of disturbance can cause the whole system to malfunction. Therefore it is required for that system to minimize the error of disturbance for increasing the efficiency. Therefore from those figures mentioned above, where we use an observer based robust adaptive control technique to minimize unwanted disturbances, the waveforms shows the convergence of the errors to zero, making the system perform effortlessly.

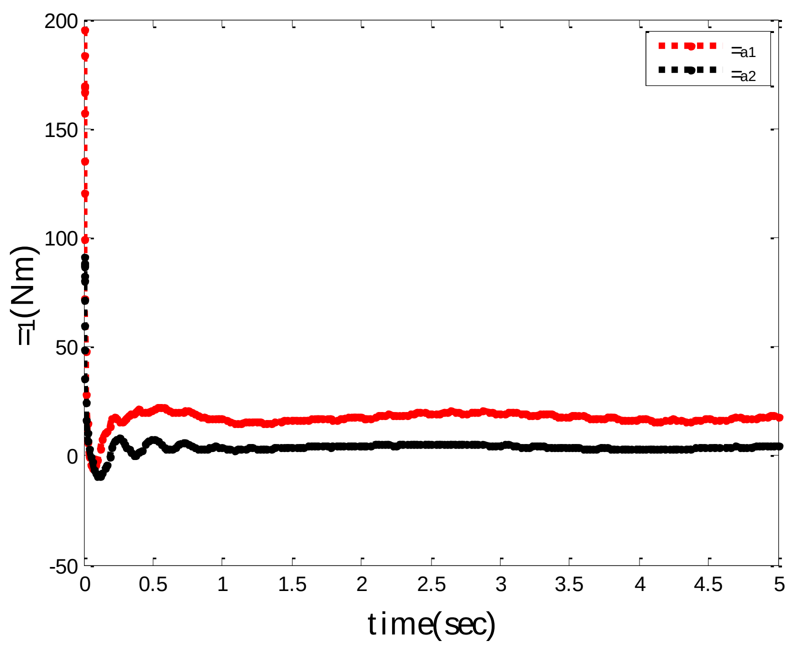


Figure 6-20 Control input of robot manipulator 1.

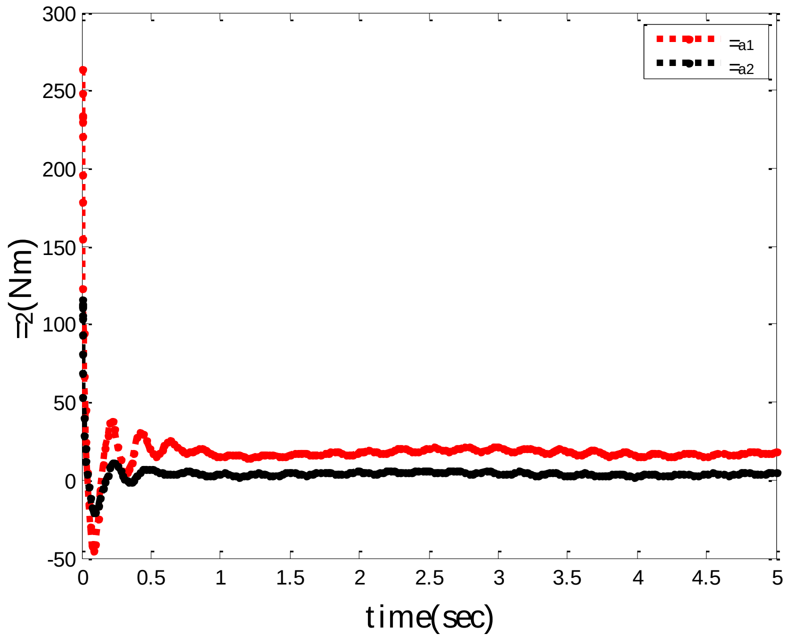


Figure 6-21 Control input for robot manipulator 2.

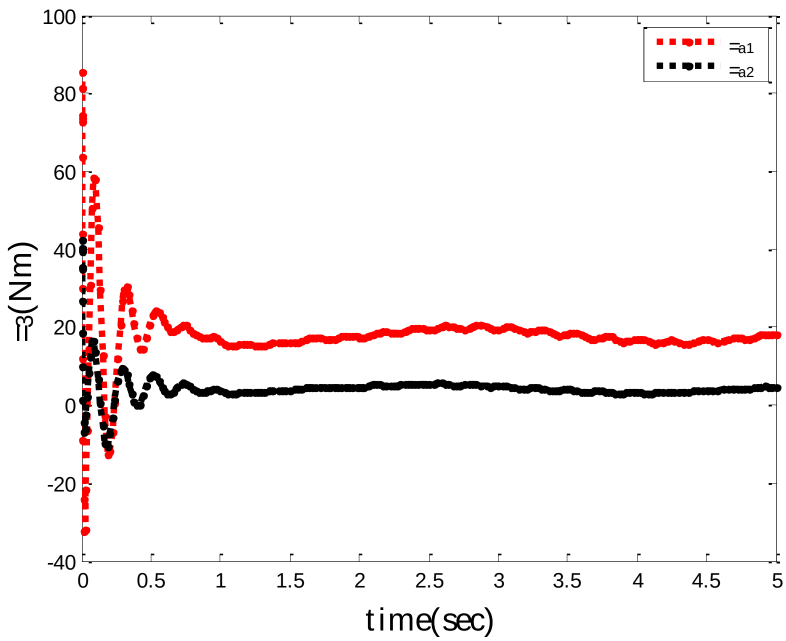


Figure 6-22 Control input for robot manipulator 3.

Figure. 6-18, 6-19, 6-20 shows the control effort required by each robot manipulator, if we go back and compare it with a single robot manipulator, the control effort required for synchronization purpose is high at the initial stage, when compared to a robot manipulator working alone. Here we can see that the first robot manipulator requires around 200 Nm of torque to reach the defined bound, the second manipulator initially takes around 270 Nm of torque to reach the prescribed bound, followed by 90 Nm required by the third robot manipulator. The control effort required at the start by the third manipulator is same when a single robot manipulator working alone.

6.9 Summary

In this chapter we have considered the synchronization problem of multi robot manipulators using observer-based robust adaptive control scheme with time-varying parametric uncertainties, unmodeled perturbations and external force disturbances included in the dynamical equation of the manipulator systems. It has been shown that the proposed control scheme developed for three identical robot manipulator system with two degree of freedom can coordinate manipulator articulations to track given time-varying trajectory. The obtained simulation results from the multiple robot manipulator model demonstrates the effectiveness of the observer based robust adaptive approach with robust tracking control also the boundedness of all closed-loop signals and the convergence of tracking error are proved to be achieved.

CHAPTER 7

CONCLUSIONS

In this thesis we present an observer based robust adaptive tracking control for the compensation of modelling errors, time-varying parametric uncertainties, un-modeled perturbations and external force disturbances considered in the dynamical equations for a class of robot manipulator. Without any prior knowledge about the bound of uncertainties and disturbances, an adaptive algorithm is proposed with time invariant and time-varying high gain observers to achieve robust tracking. The stability of these two control schemes are proved through Lyapunov method. The observer based robust adaptive control scheme is such that it only requires to measure the angular position. The boundedness of all closed-loop signals and the convergence of tracking error are proved. Then the response between time invariant and time varying observer gains are compared using like design parameters. The result shows observer designed with time variant gains exhibited smooth and faster response with larger control effort. The simulation results are presented to illustrate the effectiveness of the method.

Later in the thesis the synchronization problem in identical multiple robot manipulators is considered under cooperative schemes. The main aim here is synchronize each robot manipulator under the influence of uncertainties and external force disturbances. Computer simulations are carried on three identical two degree of freedom robot manipulators based on the given control law. Simulation results demonstrate the proposed synchronized observer based robust adaptive control technique is the most effective and superior.

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